A Presentation on

Lasing Processes, Transitions and Gain

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OBJECTIVES

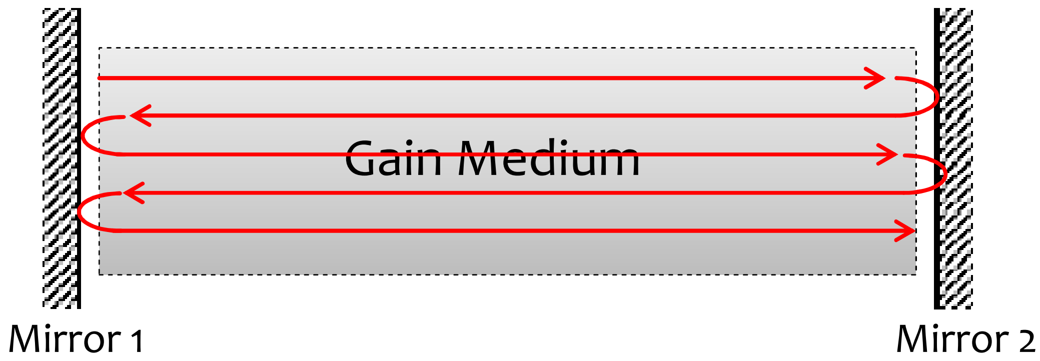
Understanding:

- **the stimulated emission**
- **pumping processes involved**
- **rate equations**
- **laser gain**

Lasers:

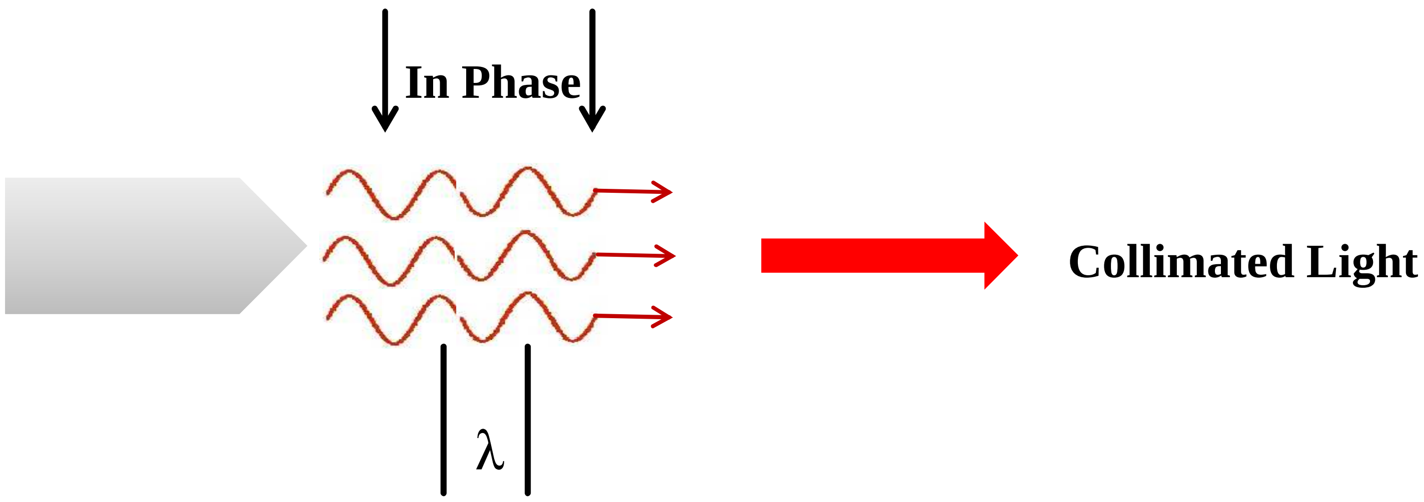
Light Amplification by Stimulated Emission of Radiation

Step1: Build a cavity
Step 2: Insert a gain medium → atoms in liquid, solid or gas
Step 3: Start the process



Properties:

- Coherence
- Monochromaticity
- Collimation



Energy Transfer Processes:

- **Spontaneous emission:**

– electron ‘naturally’ falls down from level 2 to level 1

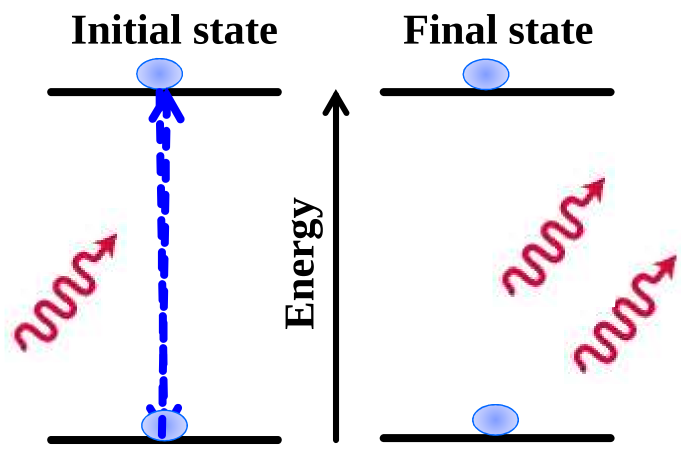
- **Stimulated emission:**

– electron is ‘triggered’ to fall down in presence of photons of energy equal to the energy difference of the initial and final state.

- **Stimulated absorption:**

– electron is raised from 2 to 1.

Amplifying Process



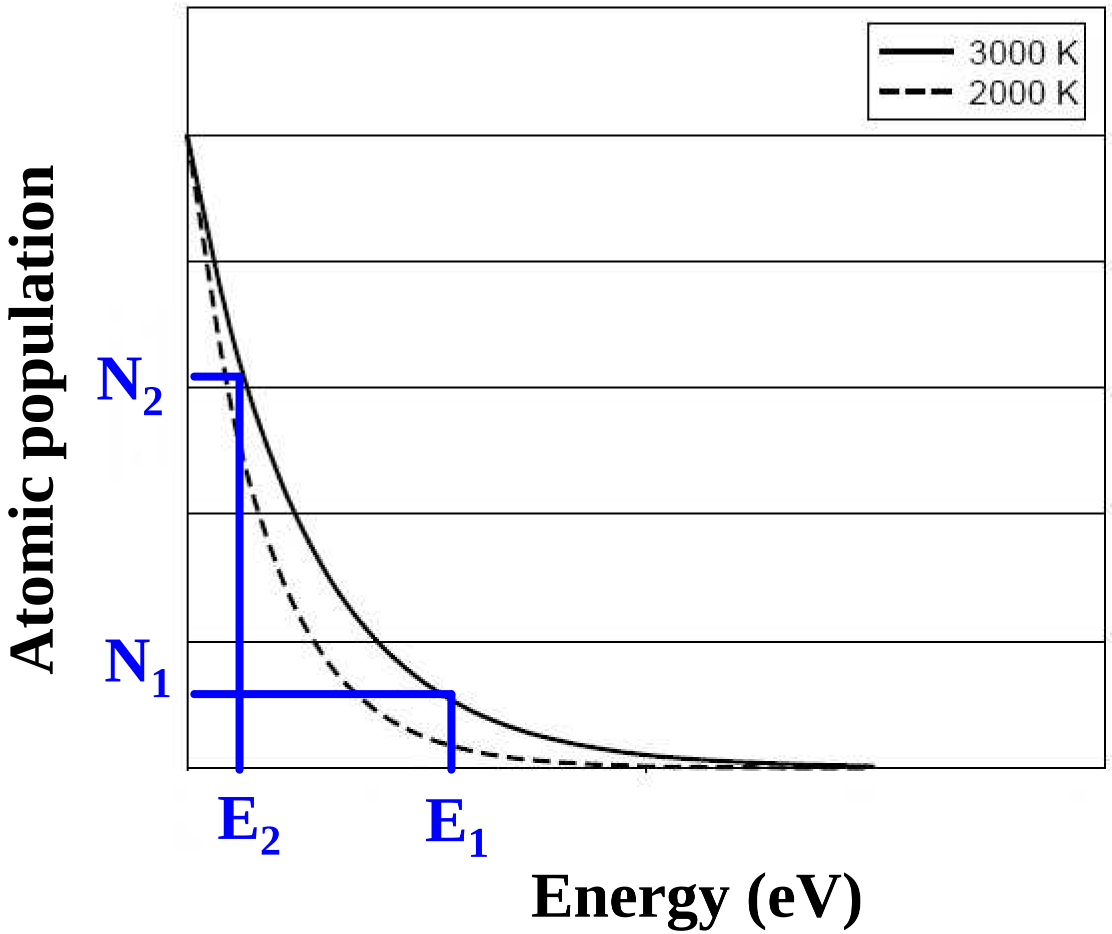
Spontaneous Emission

Spontaneous Absorption

Stimulated Emission

Thermal Equilibrium

- Atomic population is governed by temperature without an external source of energy.
- Exponential drop in population with energy.



Creating an Inversion

- Boltzmann's Distribution:

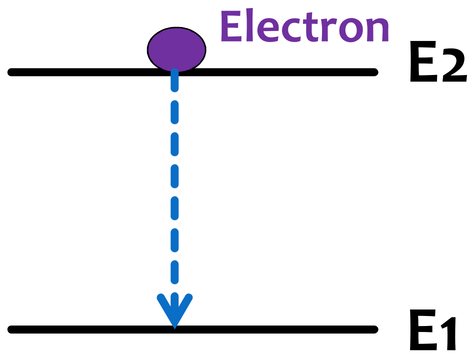
$$N_2/N_1 = e^{-\Delta E/k_B T}$$

So, cannot make laser from a system in thermal equilibrium. Need a pump.

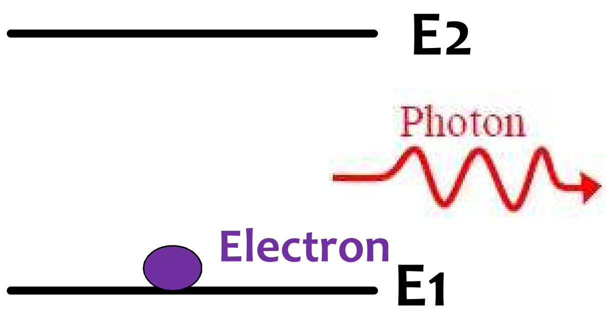
Pumping: supplying energy to the laser medium to excite the upper energy levels.

Spontaneous Emission

- Before:



- After:

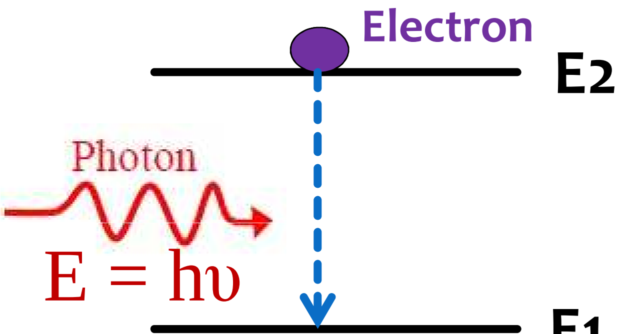


$$r_{spontaneous} = A_{21} N_2$$

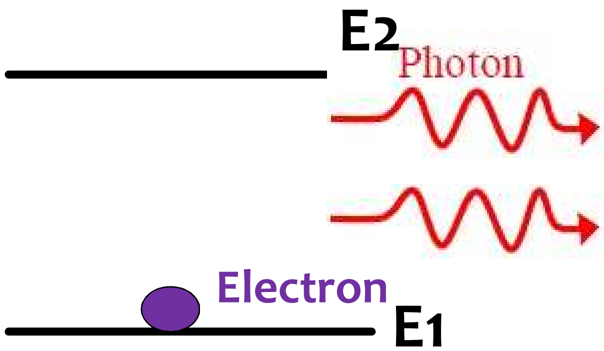
A_{21} = Einstein's coefficient
N_2 = Population at the upper level

Stimulated Emission

- Before:



- After:

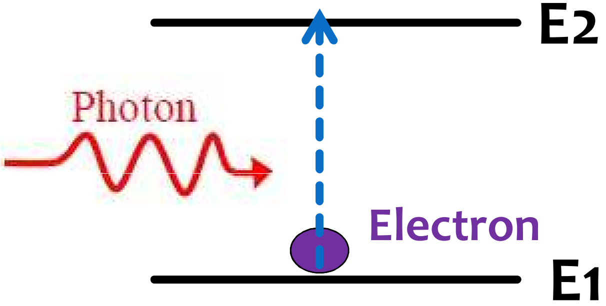


- $r_{stimulated} = B_{21}\, N_2\, \rho$

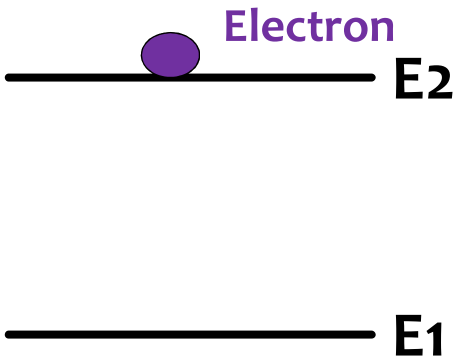
B_{21} = Einstein's coefficient
ρ = Energy density of EM wave

Stimulated Absorption

- Before:



- After:



- $$r_{absorption} = B_{12}\, N_1\, \rho$$

B_{12} = Einstein's coefficient
N_1 = population of lower level
ρ = Energy density of EM wave

Rate Equations

- For the upper level:

$$r_{absorption} = r_{spontaneous} + r_{stimulated}$$

$$B_{12}\, N_1\, \rho = B_{21}\, N_2\, \rho + A_{21}\, N_2$$

Thus,

$$\boxed{\rho = \frac{A_{21}}{B_{12}\left(\frac{N_1}{N_2}\right) - B_{21}}}$$

Rate Equations (Cont.)

- From Plank's law:

$$\rho = \frac{8\pi h\upsilon^3}{c^3}\frac{1}{e^{-\Delta E/kT} - 1}$$

$$\frac{N_2}{N_1}$$

Equating,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3}$$

Necessary Conditions for Lasing:

- $$\frac{r_{stimulated}}{r_{spontaneous}} = \frac{c^3 \rho}{8\pi h \upsilon^3}$$

For the stimulated emission to overcome spontaneous emission, ρ must be high.

$$\frac{r_{stimulated}}{r_{absorption}} = \frac{N_2}{N_1}$$

For the lasing emission to overcome absorption, $N_2 > N_1$: ***Population Invertion***

Sufficient Condition for Lasing:

- $\boxed{\text{Gain} > \text{Loss}}$

Gain coefficient: power gain per unit length

$$g = \frac{\Delta P}{P \Delta x}$$

Losses:

- Transmission at the mirror
- Absorption and Scattering at the mirrors
- Absorption in the laser medium

Threshold Gain:

Threshold Condition: $\boxed{\text{Gain = Losses}}$

Lets,

g = gain coefficient

γ = effective loss coefficient

L= length of the active medium

R_1, R_2 = reflectances of mirrors M_1 and M_2

Threshold Gain (cont.):

In travelling from M1 to M2, the beam power increases from P_0 to P:

$$P = P_0 e^{(g-\gamma)L}$$

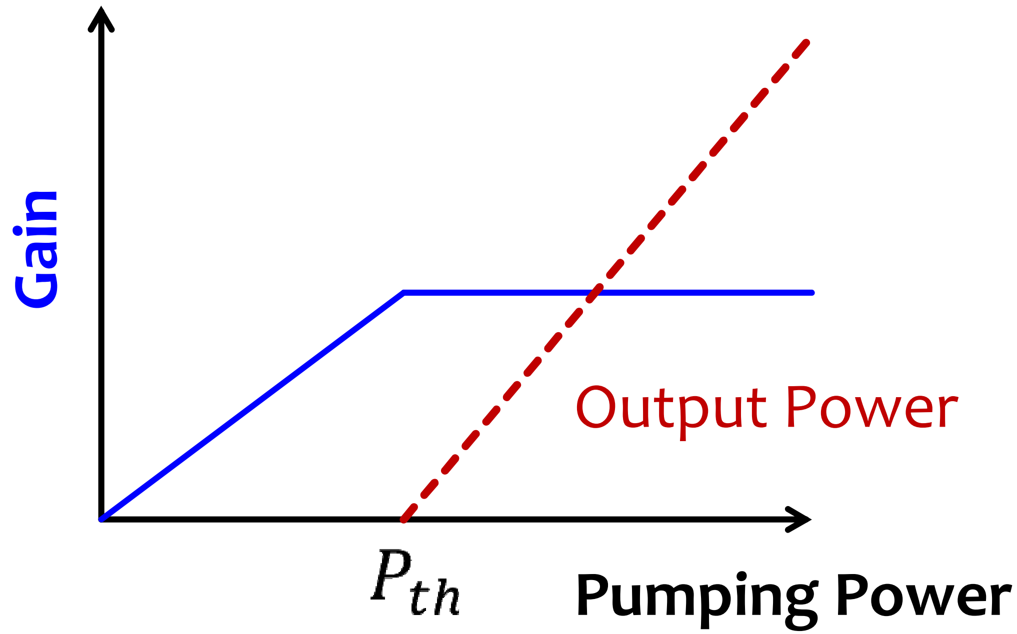
After a complete round trip:

$$G = \frac{Final\ Power}{Incident\ Power} = R_1 R_2 e^{2(g-\gamma)L}$$

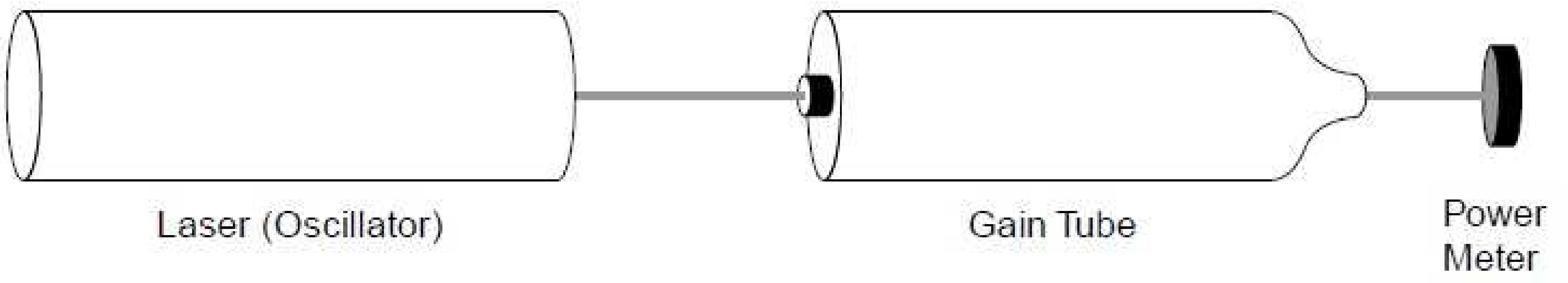
For threshold: $\mathbf{G = g_{th} = 1}$

$$g_{th} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

Pumping Threshold:



Experiments to Measure g_{th}:



MOPA Configuration

- Selecting the laser wavelength, gain for any transition can be calculated
- Threshold gain can be calculated to determine minimum required reflectivities or maximum inserted loss.

FWHM Linewidth:

- Distance between the half power points of the gain spectrum

- The primary reason of broadening: Doppler Effect: $\upsilon = \upsilon_0(1 \pm \frac{v}{c})$
- FWHM linewidth due to Doppler effect:

$$\Delta\upsilon = 2\upsilon_0 \sqrt{(\frac{2kT \ln(2)}{Mc^2})}$$

Pumping: A closer look

- Pumping must be **Selective** $\longrightarrow$ upper lasing level is populated, lower lasing level empty.

- The excitation process can be

a) Electrical: e.g., electrical discharge in He-Ne laser
b) Optical: e.g., flashlamp in Ruby and Nd:YAG laser
c) Chemical and/or nuclear

The emission spectrum of the lamp should have a reasonable value at the absorption peak of the laser medium.

Pumping: A closer look

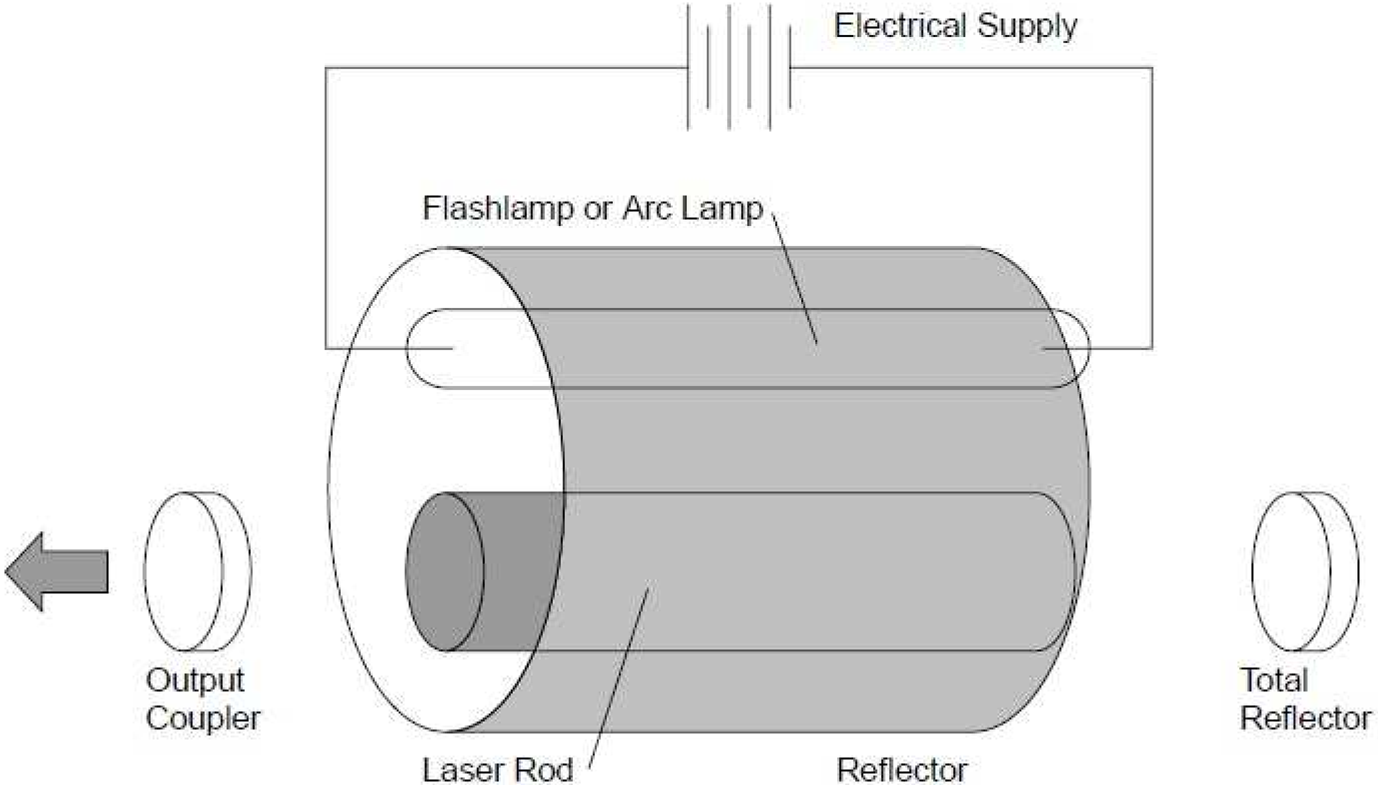
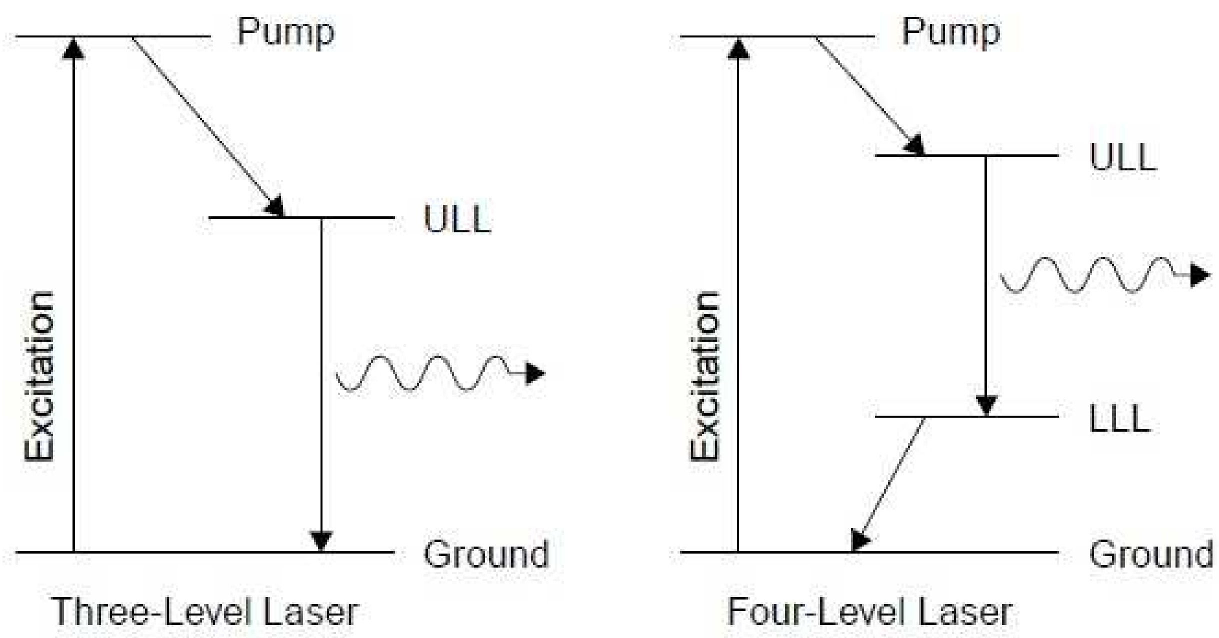


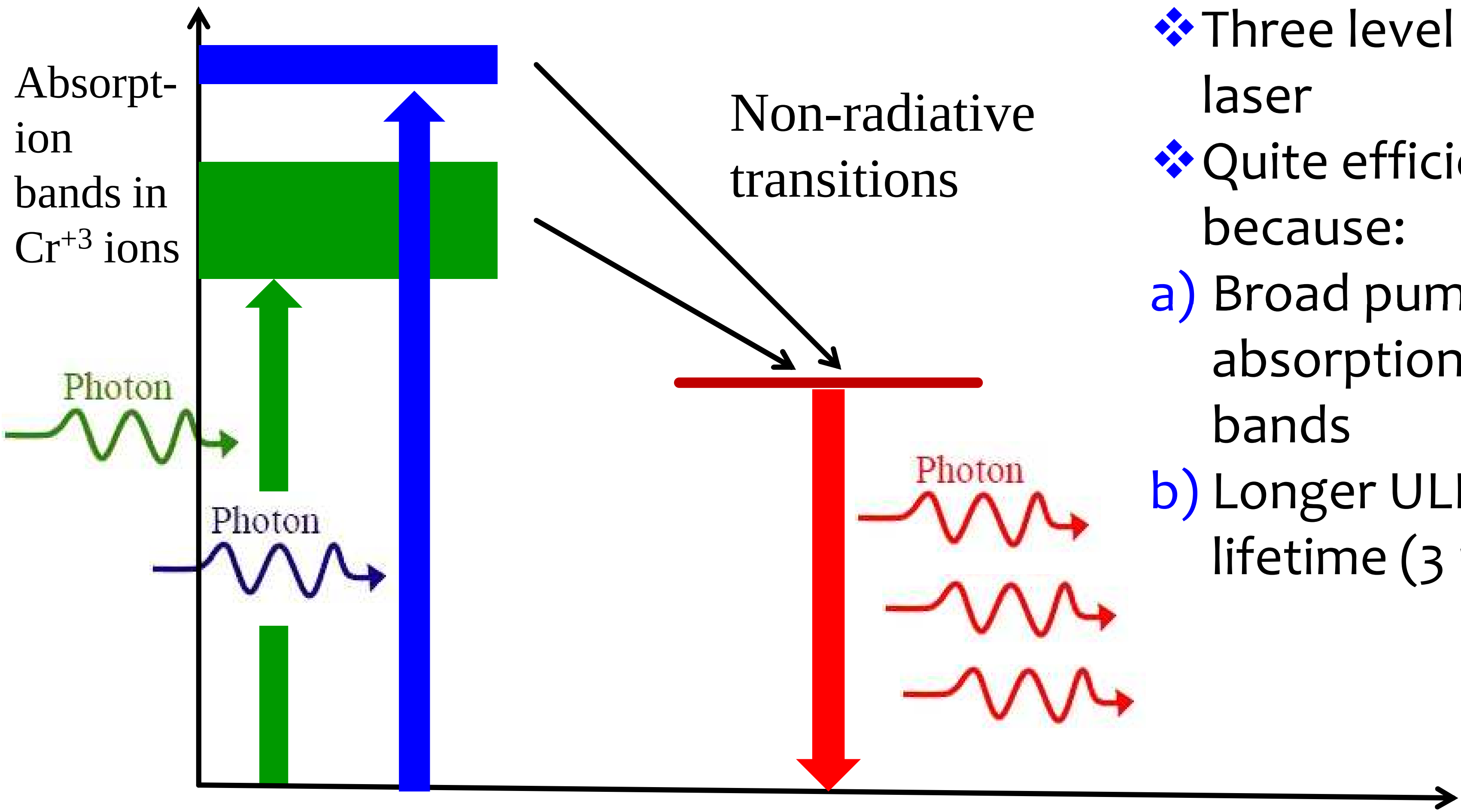
Fig. : Optical Pumping for Lasers.

Three & Four Level Laser:

- Three level lasers requires higher pumping power.
- There is a time delay for the buildup of population in 3-level laser, whereas in 4level, the population inversion occurs almost immediately.
- 4-level laser more efficient



Ruby Laser



- Three level laser
- Quite efficient because:

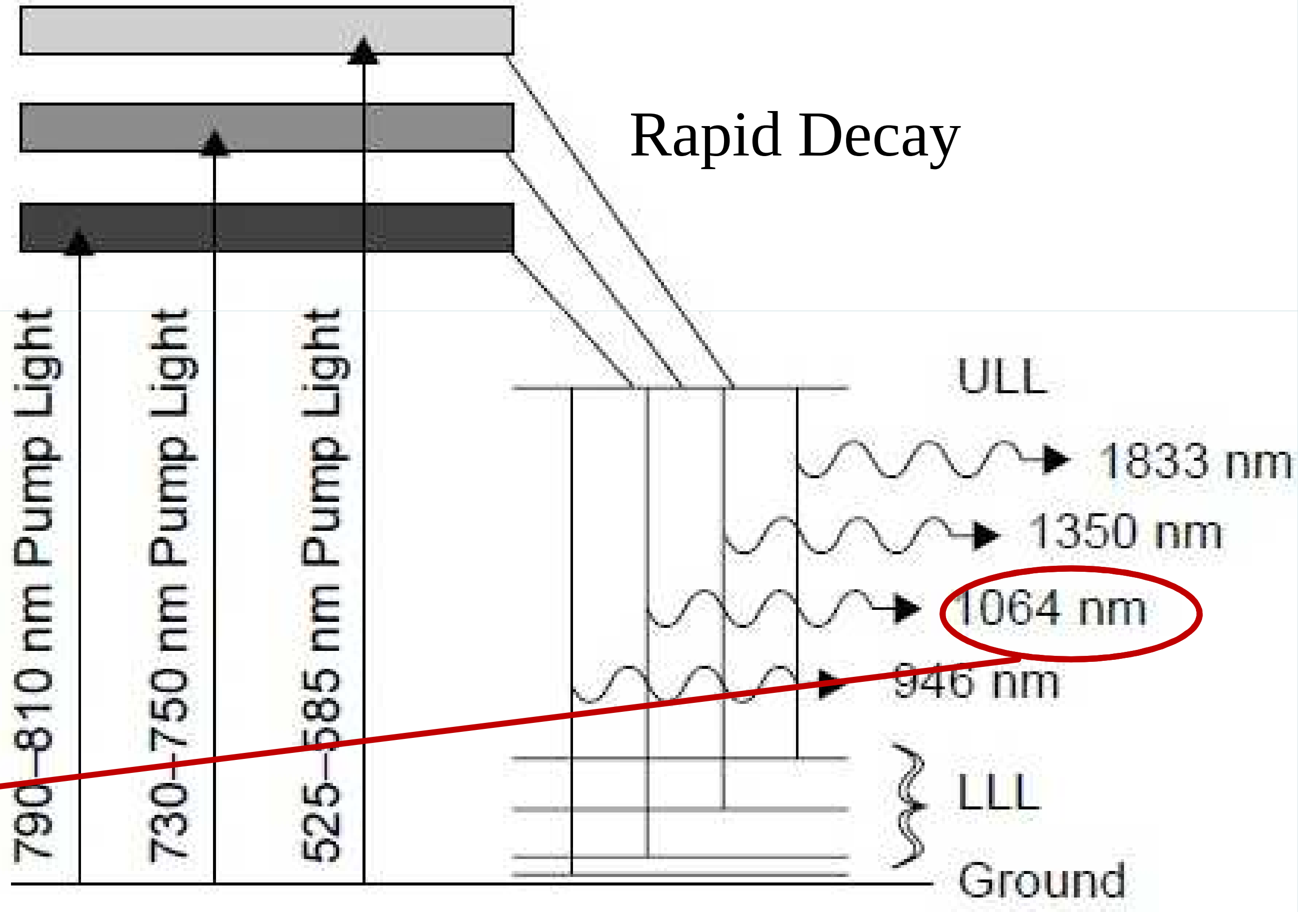
a) Broad pump absorption bands

b) Longer ULL lifetime (3 ms)

Nd:YAG Laser

- 4 level laser
- Multiple pump bands

Most powerful emission

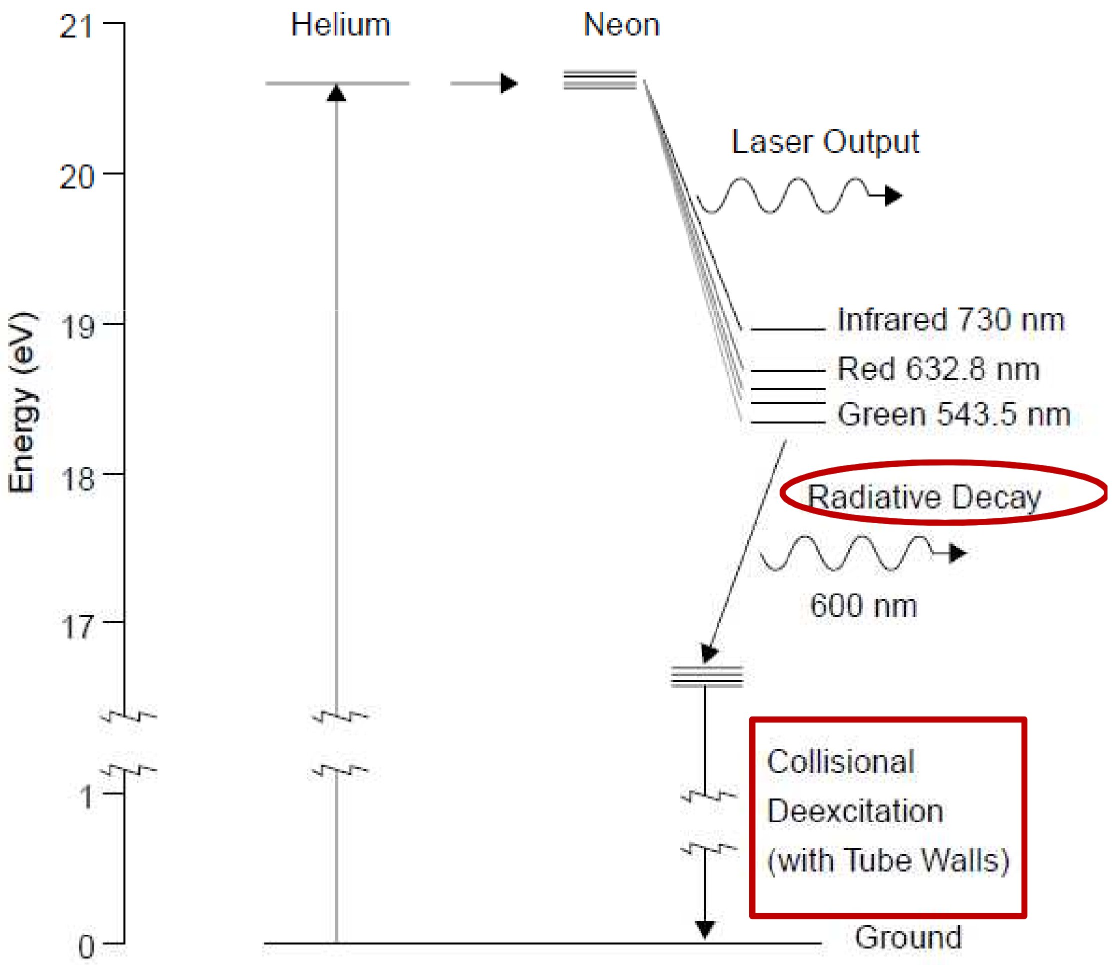


Nd:YAG Laser System

He-Ne Laser

- Electrical discharge excites He ions by collisions.
- Excited He atoms collide with Ne atoms→ transfers atoms to the ULL
- No resonant energy level of He at the energy level value of the LLL of Ne → No transfer of energy

Does Not contribute to laser output: seen as orange line in spectrum



CW and Pulsed Lasing Action

- LLL lifetime > ULL Lifetime
- ULL filled very quickly, but eventually LLL population will exceed ULL population and Lasing will cease.

$\Downarrow$

Pulsed Mode Operation

Uses: marking, cutting, drilling, range finder, surgery.

- LLL Lifetime < ULL Lifetime $\Downarrow$

CW operation

$$r_{spontaneous} = A_{21} N_2$$

where

$$A_{21} = 1/\tau$$

So, τ increases, A_{21} decreases, probability of spontaneous emission decreases.

N2 Laser: Pulse Mode Operation

- $\tau_{ULL} = 10\ ns, \tau_{LLL} = 10\ ms$

A fast pumping mechanism (current $\approx 1000\text{A}$) to fill ULL

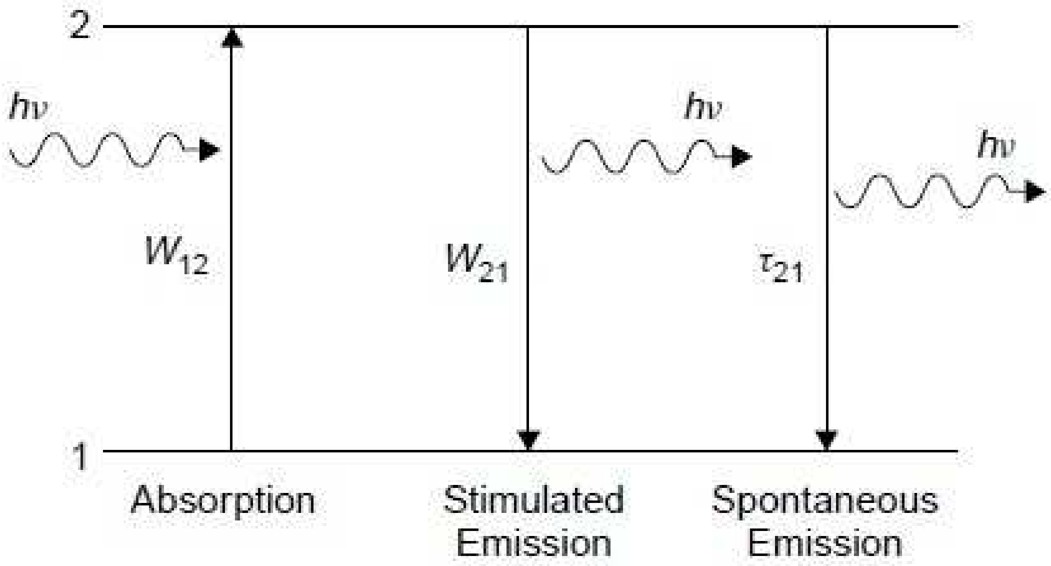
Lasing ensues until at 10 ns

After about 10 ms the LLL depopulates.

Thermal Population Effects:

- Effect of thermal population: almost negligible
- Where two or more transitions are possible, with one having a lower level close to ground state, a transition with a higher lower level may be favored

Rate Equation for 2-level system



Level 2:

$$\frac{dN_2}{dt} = Abs._{1-2} - Stim.Em._{2-1} - Spon.Em._{1-2}$$

$$= W_{12}N_1 - W_{21}N_2 - N_2/\tau_{21}$$

Level 1:

$$\frac{dN_1}{dt} = W_{21}N_2 - W_{12}N_1 + N_2/\tau_{21}$$

2-level System (Cont.)

- Rate of change of population:

$$\frac{d\,\Delta N}{dt} = \frac{dN_1}{dt} - \frac{dN_2}{dt}$$

Equating to zero (at s.s) and putting W_{12}=W_{21}, we get:

$$\Delta N = \frac{N_0}{1 + 2W_{21}\tau_{21}}$$

W_{21}increases$\longrightarrow$ ΔN tends to zero, thus N1 and N2 will highest be equal: **No population Inversion**

3 Level System

- Pump level:

$$\frac{dN_3}{dt} = W_{13}(N_1 - N_3) - \frac{N_3}{\tau_3}$$

Spontaneous decay from 3→2 and 3→1

Equating to zero for s.s:

$$W_{13}N_1 = N_3\left(\frac{1}{\tau_3} + W_{13}\right)$$

- Practically, the decay from the pump level to the upper level must be much faster than the decay from the upper to the lower lasing levels.

$$W_{13}N_1\tau_3 = N_3$$

3 Level System(Cont.)

- Level 2:

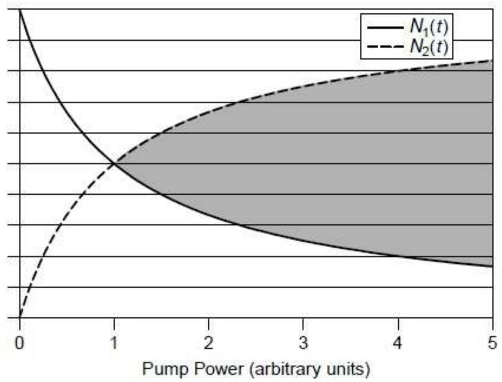
$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}$$

Equating to zero:

$$N_2 = N_3 \frac{\tau_{21}}{\tau_{32}}$$

Assuming $\tau_3 = \tau_{32}$:

$$\Delta N = N_1(W_{13}\tau_{21} - 1)$$



A minimum pumping rate equal to $W_{13}\tau_{21}$ is needed just to get half of the population at ground state to the upper lasing level.

Four Level laser:

- Pump level:

$$\frac{dN_4}{dt} = W_{14}(N_1 - N_4) - \frac{N_4}{\tau_4}$$

Equating to zero and assuming $N_1 \ll N_4$:

$$W_{14}\tau_4 N_1 = N_4$$

Four Level laser (cont.):

- Level 3:

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3}$$

Equating to zero:

$$N_3 = N_4 \frac{\tau_3}{\tau_{43}}$$

Four Level laser (cont.):

- Level 2:

$$\frac{dN_2}{dt} = \cancelto{0}{\frac{N_4}{\tau_{42}}} + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}$$

Equating to zero:

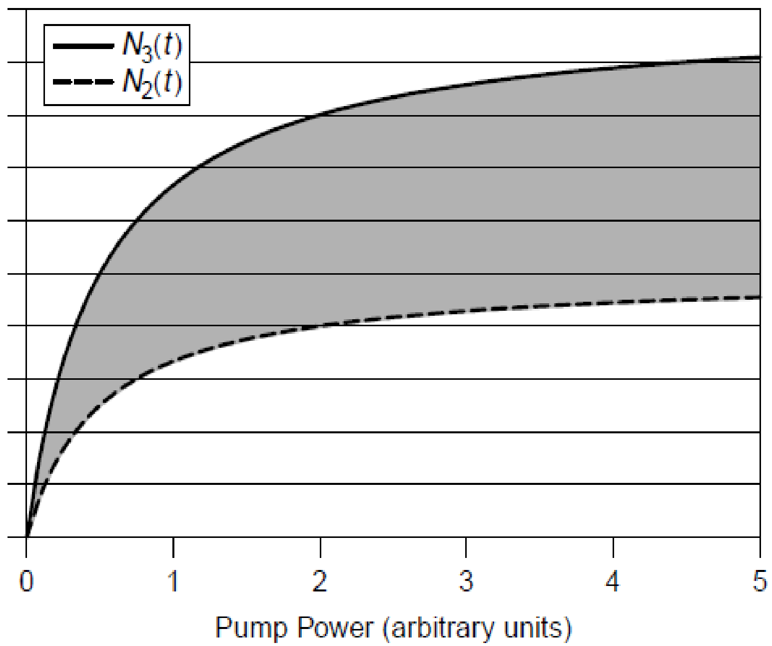
$$N_2 = N_3 \frac{\tau_{21}}{\tau_{32}}$$

Four Level laser (cont.):

- Population Inversion:

$$\Delta N = N_1 W_{14} \tau_3$$

Thus, inversion occurs when any pump energy is supplied.



Gain:

- Gain Coefficient $g = (N_2 - N_1)\sigma_0$

σ_0= cross section of the stimulated emission process

- $\sigma(\upsilon) = Sg(\upsilon) = \frac{\lambda^2}{8\pi t_{sp}} g(\upsilon)$

$g(\upsilon)$ is the lineshape of the gain function and S is the transition strength or oscillator strength.

- For homogeneous processes:

$$g(\upsilon_0) = \frac{2}{\pi\Delta\upsilon}$$

$\Delta\upsilon$ is the FWHM linewidth.

- For inhomogeneous processes:

$$g(\upsilon_0) = \frac{1}{\Delta\upsilon}$$

Saturation:

- **The previous calculation don't account for the stimulated emission processes.**

- If Light present ➔ Stimulated Emission (N_2W_{21}) ➔ Loss
- Also a absorption process (N_1W_{12}) from LLL to ULL.
- Photon flux increases ➔ amount of inversion decreases ➔ gain decreases

Saturation (cont.):

- No saturation:

$$P_{output} = P_{input} e^{g_0 l}$$

- Light present in the cavity, gain medium behaves as a 'saturated amplifier':

$$P_{output} = P_{input} + \ g_0 l$$

Efficiency:

- $\eta_{Optical}$: conversion of electrical energy to optical energy → depends on lamp technology
- $\eta_{Coupling}$:Coupling of the pump light to laser medium → depends on geometry of laser medium
- $\eta_{Absorption}$: depends on the wavelength of the source and the absorption spectrum of gain medium

Efficiency (cont.):

-
- $\eta_{Quantum} = \frac{E_{ULL} - E_{ground}}{E_{pump} - E_{ground}}$ → depends on the particular atomic medium

Overall Efficiency: Typically 1% for gas lasers

$$\eta_{pump} = \eta_{Optical} \cdot \eta_{Coupling} \cdot \eta_{Absorption} \cdot \eta_{Quantum}$$

Required Pump Power:

- The minimum pump energy required:

$$P_{minimum} = \frac{dN_{ULL}}{dt} V h\upsilon_{mp}$$

Active Volume of the gain medium

Photon energy required at the ULL

Now,

$$N_{ULL}(t) = N_{ULL}(0)\exp\left(-\frac{t}{\tau}\right)$$

Differentiating

$$\frac{dN_{ULL}}{dt} = -\frac{t}{\tau}N_{ULL} \approx -\frac{t}{\tau}\Delta N$$

For 4 level system

Output Power from a Laser:

- Threshold gain: Round trip gain = Round trip loss

$$g_{th} = \gamma + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

- After gain reaches threshold, it saturates to at equilibrium

$$g_{sat} = \frac{g_0}{1 + 2\, I/I_{sat}}$$

Unsaturated gain of the amplifier
$g_0 \propto \Delta N$

Output Power from a Laser: (cont.)

- Solving for Intensity I,

$$I = \frac{\left\{\frac{2g_0}{[2\gamma l \ln(1/R_1R_2)]} - 1\right\} I_{sat}}{2}$$

- By multiplying area, We get the power available.
- g_0 can be determined experimentally, thus maximum power can be calculated.


THANK YOU!