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Course Title: Laser Theory

Lasing Processes, Transitions and Gain

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Chapter 4

Lasing Processes

To understand the laser emission, we have to indulge ourselves in understanding the basic principles of laser transition, gain and sustaining the laser oscillation inside the laser cavity. In this chapter, we analyze the basic principles and processes involved in laser emission.

The word 'laser' is an acronym for "Light Amplification by Stimulated Emission of Radiation". After the first demonstration of laser in 1960, its development has been extremely rapid. To understand the basic lasing setup, helium–neon (He-Ne) laser can be taken as an example. It is one of the first few demonstrated lasers which is used commonly in holography and bar code scanning.

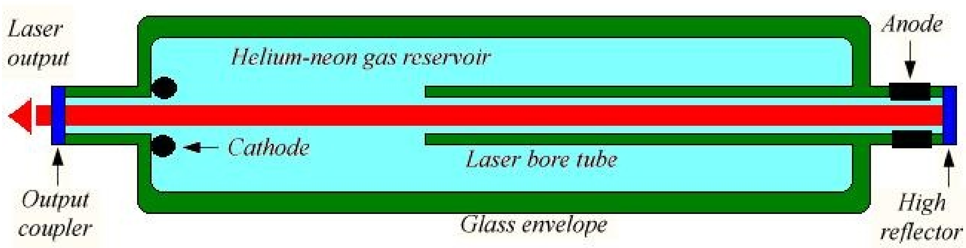

Figure 4.1: Elements of a helium-neon laser tube.

The structure of a typical He-Ne laser is illustrated in Figure 4.1. The basic laser consists of a glass tube filled with helium and neon gases in a ratio of about 10 parts helium to 1 part neon. When the electrodes are energized by a high-voltage power supply, electrical discharge occurs between the anode and cathode through a capillary tube called *plasma tube*. At either end of the laser are cavity mirrors, one of which is fully reflecting, and the other partially reflecting. The small portion of light (typically around 1%) that is transmitted through the front mirror is the actual laser beam itself.

4.1 Characteristics of Coherent Light

Laser sources are unique in the sense of three special properties that lead to their usefulness in many applications. As depicted in Figure 4.2, coherence, monochromaticity, and collimation (directionality) are the three properties.

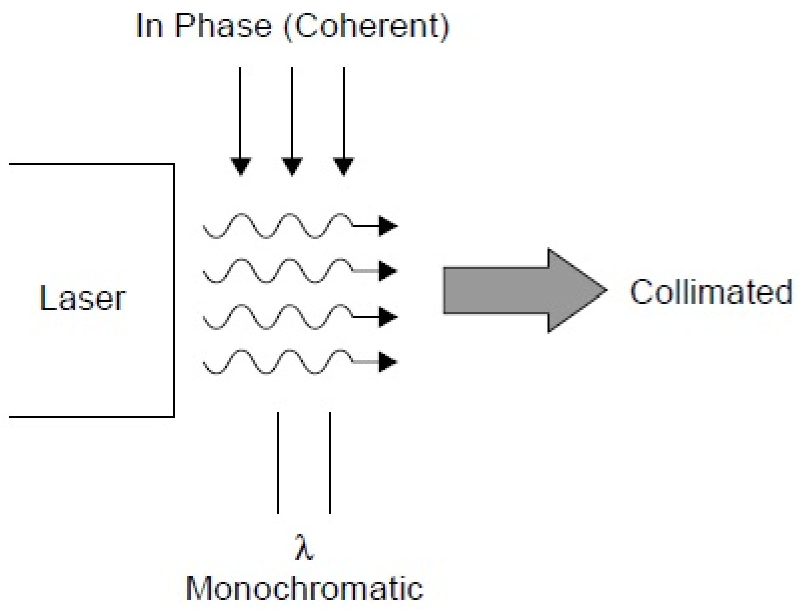

Figure 4.2: Properties of laser light.

- **Coherence:** Coherent light are light waves that are "in phase" with one another. To stay in phase it is required that all emitted photons are at exactly the same or almost same wavelength. If some photons are at a different wavelength, the phase of those photons relative to others will be different and the light will not be coherent. They must also be highly directional i.e., all moving in the same direction.

- **Monochromaticity:** Its the ability of the laser to produce light that is at one well-defined wavelength. It is required for coherence since photons of varying wavelengths cannot be coherent.

- **Collimation:** Collimated light rays are parallel, and therefore will spread slowly as they propagate. This property of laser makes it possible to use the laser as a level in construction or to pinpoint speeders on a highway.

4.2 Boltzmann Distribution and Thermal Equilibrium:

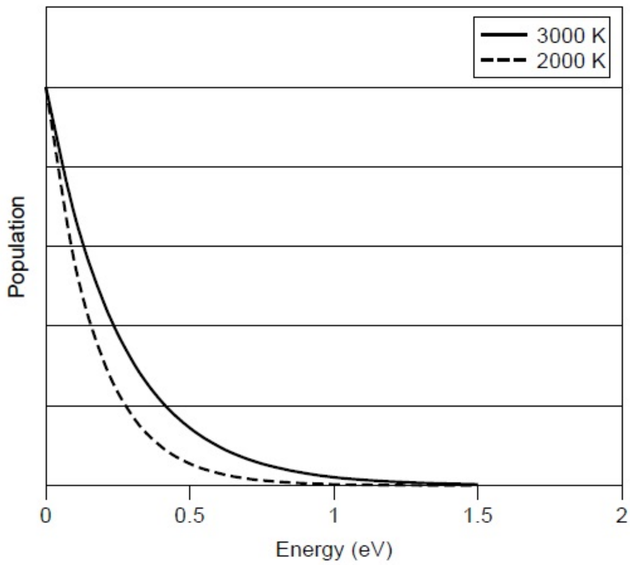

Figure 4.3: Boltzmann distribution of atomic energies at various temperatures.

The Boltzmann distribution which applies to a system without an external source of energy predicts the population of atoms at a given energy level as follows:

$$N = N_0 \, exp\left(-\frac{E}{kT}\right), \tag{4.1}$$

where N is the population of atoms at the given energy level, N_0 the population of atoms at ground state, E the energy above ground level, k is Boltzmanns constant, and T is the absolute temperature. Such a system is said to at *thermal equilibrium* and the population distribution is shown in Figure 4.3. If temperature is increased, more atoms will reach higher energies. Many atoms at high energies will then drop to a lower-energy state and in doing so, emit a photon of light with a wavelength corresponding to the energy difference between the states. In such a system, there are a large number of upper and lower levels, and these levels may span a range of energies so that the output from such a source is broadband.

4.3 Attainment of a Population Inversion:

For achieving lasing action, the population at the higher energy state should be greater than population at the lower energy state. This non-equilibrium condition can be achieved by supplying energy to excite atoms into the upper energy level. This excitation process is called *pumping*. It may be accomplished by any number of means, including electrical, thermal, optical, chemical, or nuclear.

- A laser utilizing electrical pumping uses an electrical discharge to excite atoms directly to high energy levels. In some lasers, the actual lasing atom is pumped with energy via a multi-step process involving the transfer of energy between two atomic species. For example, in the He-Ne laser, the actual lasing atom is neon, which is pumped with energy through collision with a helium atom which is excited by the electrical discharge.

- A laser utilizing optical pumping uses light from a flash lamp or an arc lamp to excite atoms to higher energy state. Ruby laser and Nd:YAG laser (neodymium ions in a yttrium-aluminum-garnet host glass) use optical pumping.

Pumping in He-Ne Laser: An Example

One of the main concerns of the pumping process is that the pumping source or method has to ensure that the upper level is pumped selectively while the lower level is not. Let us consider, the He-Ne gas laser which employs electrical discharge as pumping source. The discharge excites helium atoms by electron collisions. Some excited helium atoms (with a

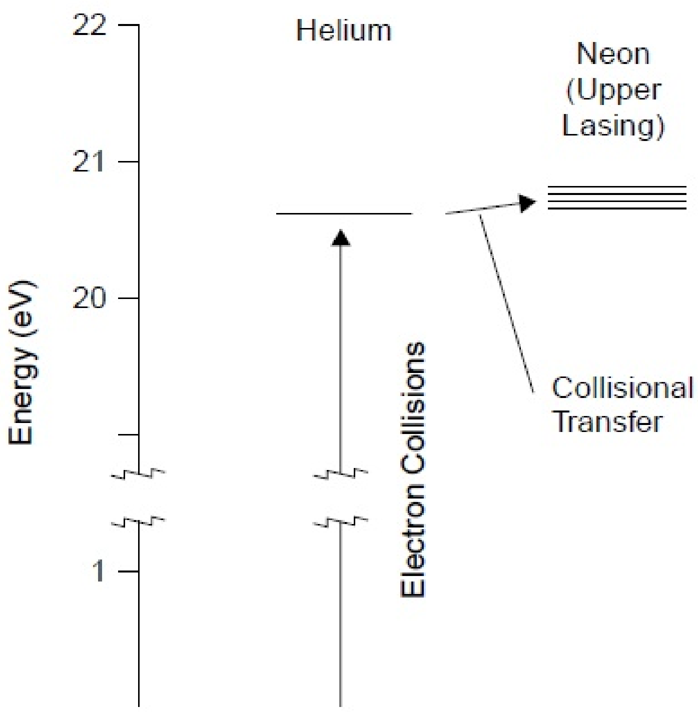

Figure 4.4: Pumping mechanism in the heliumneon laser.

specific energy of 20.61 eV) can collide with neon atoms and in doing so transfer energy to them at an almost identical energy level ($2p^5 5s^1$ at 20.66 eV). The energy levels involved are depicted in Figure 4.4. In actual case, as neon is a multi-electron atom, the $2p^5 5s^1$ splits into 4 levels. Helium lacks a level close to the $2p^5 3p^1$ level of neon, so it does not receive energy from collisions with helium. Thus, a population inversion is created between the levels $2p^5 5s^1$ and $2p^5 3p^1$ of Ne, which act as upper and lower laser levels,respectively, for the 632.8 nm red transition. The population at these levels are plotted at thermal equilibrium and in an operating laser in Figure 4.5.

4.4 Stimulated Emission

If an electron is in excited state, it may return to the ground state with the emission of photon. The emission process can occur in two distinct ways. These are: (a) the *spontaneous emission* process in which the electron drops to the mower level in an entirely random way (Figure

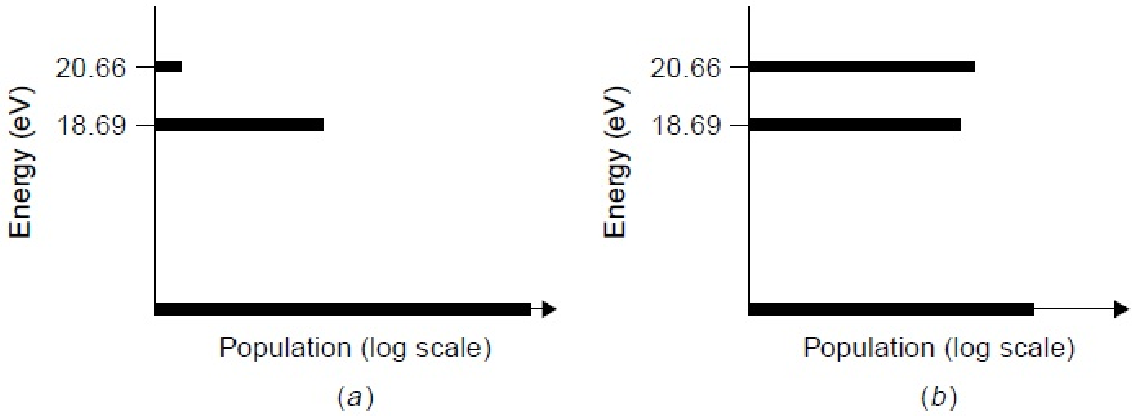

Figure 4.5: Population at the $2p^5 5s^1$ (with an energy of 20.66 eV) and $2p^5 3p^1$ (with an energy of 18.69 eV) levels of neon atoms (a) at equilibrium and (b) in a He-Ne laser.

4.6(a)) and (b) the *stimulated emission* process in which the electron is 'triggered' to undergo the transition by the presence of photons of energy equal to the energy difference of the initial and final state (Figure 4.6(b)).

The later is essentially the laser emission process. In the stimulated emission process, the excited atom emits a photon of exactly the same wavelength and phase as the incident photon. Thus, this is a *amplifying* process. The number of atoms in the excited state must be larger than the ground state; else the incident photon will be absorbed. Hence, population inversion is needed for stimulated emission. Another consideration is spontaneous emission. Spontaneous emission between these two levels is unwanted as it reduces the population of the upper level.

4.5 Rate Equations and Criteria for Lasing

The general criteria for a net photon gain to occur is that the rate of stimulated emission must exceed that of spontaneous emission plus that of all the losses.

The rate of absorption of photons depends on the number of atoms in the lower state as well as the energy density of incident photons. Mathematically, the rate of absorption may be stated as

$$r_{\text{absorption}} = B_{12} N_1 \rho, \tag{4.2}$$

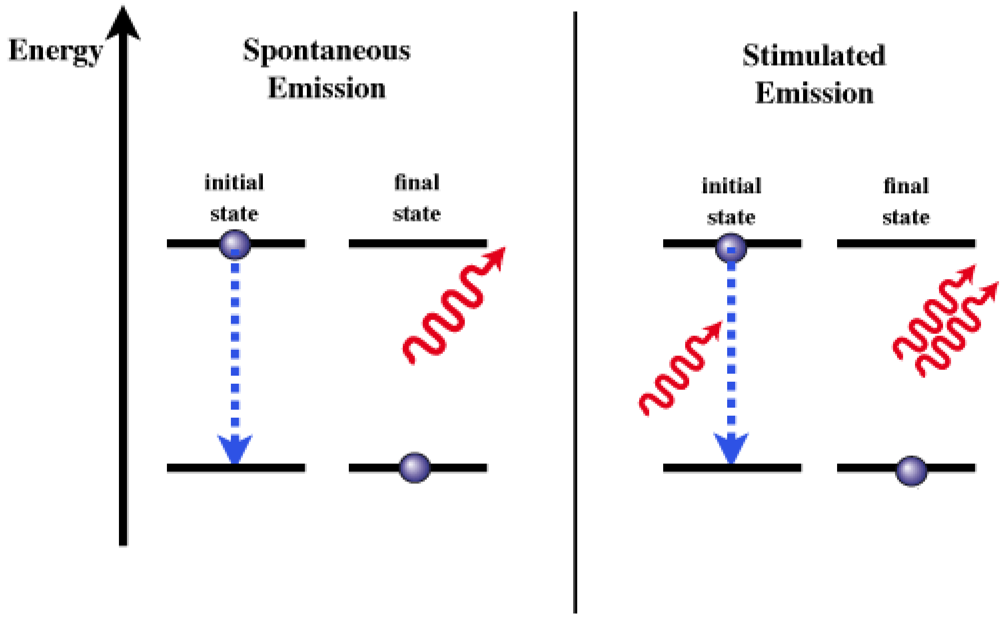

Figure 4.6: Spontaneous and stimulated emission.

where B_{12} is a proportionality constant called *Einsteins coefficient*, N_1 the number of atoms at the lower-energy state, and ρ the energy density. ρ represents the number of photons that have the exact energy for the transition between energy levels E_1 and E_2.

Similarly, the rate of stimulated emission depends on the number of atoms at the upper state and can be written as

$$r_{\text{stimulated}} = B_{21} N_2 \rho, \tag{4.3}$$

where B_{21} is *Einsteins coefficient*, N_2 the number of atoms at the upper-energy state, and ρ the energy density.

The rate of spontaneous emission does not depend on incident energy density as atoms emit photons spontaneously regardless of external conditions. It depends solely on the number of atoms at the upper energy state available to emit a photon and is given by

$$r_{\text{spontaneous}} = A_{21} N_2, \tag{4.4}$$

where A_{21} is *Einsteins coefficient* for spontaneous emission, and N_2 the number of atoms at the upper-energy state. Here, the A coefficient for absorption is related to the spontaneous lifetime as

$$A_{21} = \frac{1}{\tau}, \tag{4.5}$$

where τ is the spontaneous radiative lifetime of the upper state.

For a system in equilibrium, the upward and downward transition rates must be equal and hence we have

$$r_{\text{absorption}} = r_{\text{stimulated}} + r_{\text{spontaneous}} \tag{4.6}$$

or

$$B_{12}N_1\rho = B_{21}N_2\rho + A_{21}N_2. \tag{4.7}$$

Rearranging Equation (4.7), we get:

$$\rho = \frac{A_{21}}{B_{12}(N_1/N_2) - B_{21}}. \tag{4.8}$$

We know from Plank's blackbody radiation law for cavity radiation that

$$\rho = \frac{8\pi h\nu^3}{c^3}\frac{1}{exp(-E/kT) - 1}. \tag{4.9}$$

We also know from boltzmann's law, for two energy levels: $exp(-E/kT) = N_2/N_1$. Substituting this into Equation (4.9) we get

$$\rho = \frac{8\pi h\nu^3}{c^3}\frac{1}{N_2/N_1 - 1}. \tag{4.10}$$

Comparing Equations (4.8) and (4.10), we get

$$\frac{A_{21}}{B} = \frac{8\pi h\nu^3}{c^3}. \tag{4.11}$$

Let us consider the ration of stimulated to spontaneous emission.

$$\frac{r_{\text{stimulated}}}{r_{\text{spontaneous}}} = \frac{B_{21}N_2\rho}{A_{21}N_2} \tag{4.12}$$

Substituting for the *Einsteins coefficients*, the ratio simplifies to:

$$\frac{r_{\text{stimulated}}}{r_{\text{spontaneous}}} = \frac{c^3\rho}{8\pi h\nu^3} \tag{4.13}$$

This ratio of Equation (4.13) must be large enough in order to achieve laser emission. For this ratio to be large, the energy density of incident photons (ρ) must be high. Unless the gain of the medium is extremely high in order to create a huge flux of photons as they pass down the tube, cavity mirrors will be required to contain photons within the cavity to create further amplification.

From Equations (4.2) and (4.3), the ratio between the stimulated emission and absorption is

$$\frac{r_{\text{stimulated}}}{r_{\text{absorption}}} = \frac{N_2}{N_1}. \tag{4.14}$$

Thus, for the lasing emission to overcome the absorption, N_2 must be greater than N_1. This relation proves the necessity of population inversion for laser action.

4.6 Laser Gain

Laser gain (or optical gain) is a measure of how well a medium amplifies photons by stimulated emission. The stream of photons can be treated as an electromagnetic wave and the power of the wave increases when it travels through the tube if the rate of stimulated emissions exceeds that of spontaneous emissions.Mathematically the power increases as a function of length and can be expressed as

$$P = P_0 \, exp(gx), \tag{4.15}$$

where g is the optical gain coefficient of the laser medium and x is the distance down the tube and P_0 is the incident irradiance power. The gain coefficient, g, represents the optical power gain per unit length. Mathematically,

$$g = \frac{\Delta P}{P \Delta x}. \tag{4.16}$$

The standard model for picturing a gain medium, shown in Figure 4.7, is to view a thin slice of a medium. A stream of photons with power P enters the gain medium, which has length Δx, and exits with increased power $P + \Delta P$. Gain is proportional to the net rate of stimulated emission, which is in turn equal to the rate of stimulated emission minus the rate of absorption.

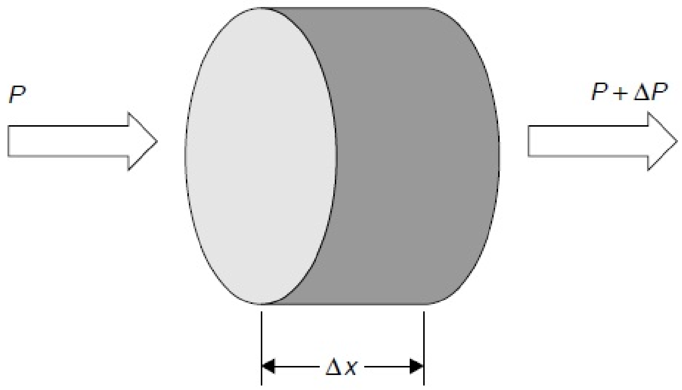

Figure 4.7: Gain in a laser medium for a length of Δx.

4.7 Linewidth

The output radiation from a laser is not actually at one single well-defined wavelength corresponding to the lasing transition, but covers a spectrum of wavelengths with a central peak as shown in Figure 4.8 and hence, have a finite bandwidth. The primary reason to broaden a line's spectral width is the *Doppler effect*. Assuming that the average speed of a gas molecule is v and that gas molecules can move in either direction at this speed, the spread in frequencies seen

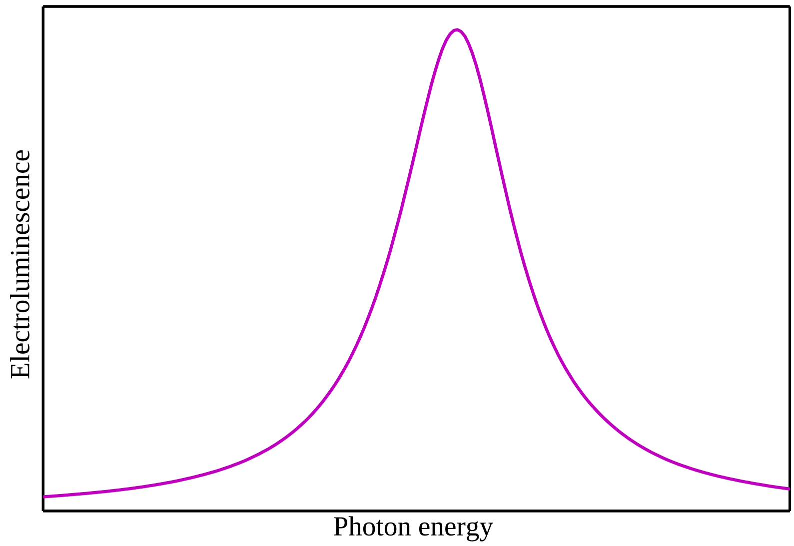

Figure 4.8: Gain curve for a practical laser.

in the output of the laser will be

$$\nu = \nu_0 \left(1 \pm \frac{v}{c}\right) \tag{4.17}$$

where ν is the output frequency of the laser, ν_0 the center frequency, and v is the average velocity of gas molecules. To calculate the average velocity, v, we can equate the kinetic energy to thermal energy as follows:

$$\frac{1}{2}mv^2 = \frac{1}{2}kT, \tag{4.18}$$

where m is the mass of the molecule, v the velocity of gas molecules, k is Boltzmanns constant, and T is the temperature. The more appropriate result can be achieved if we apply the Maxwell distribution to the velocity of the gas molecules. Using such a distribution will yield the spectral width as

$$\Delta\nu = 2\nu_0 \sqrt{\frac{2kTln(2)}{Mc^2}}, \tag{4.19}$$

where $\Delta\nu$ is the FWHM of the output, ν_0 the center frequency, M the atomic mass of the atom or molecule, and c the speed of light.

4.8 Threshold Conditions

A steady state level of oscillation is reached when the rate of amplification is balanced by the rate of loss. Thus, while a population inversion is a necessary condition, it is not a sufficient one because the minimum of the gain coefficient must be large enough to overcome the losses and sustain oscillation.

4.8.1 Laser Losses

The total loss of the system is due to a number of different processes, the most important ones include:

- Transmission at the mirror– the transmission from one of the mirrors usually provides the useful output, the other mirror is made as reflective as possible to minimize losses.
- Absorption and scattering at the mirrors.
- Absorption in the laser medium due to transitions other than the desired transitions.
- Scattering at the optical inhomogeneties in the laser medium– this applies particularly to solid state lasers.
- Diffraction losses at the medium.

4.8.2 Pumping Threshold

There is a threshold on the pumping rate that must be reached for lasing action to occur. First, until the pump rate is sufficient to allow population inversion, laser gain will not begin. But population inversion alone does not guarantee laser output, as the laser will not produce output until the gain overcomes losses in the laser. The situation is outlined in Figure 4.9, where gain is seen to increase as pumping power; however, laser output does not begin until gain in the laser is equal to the total losses. This is the threshold pumping power (P_{th}), and any increase in pumping power after that point will lead to an increase in laser output.

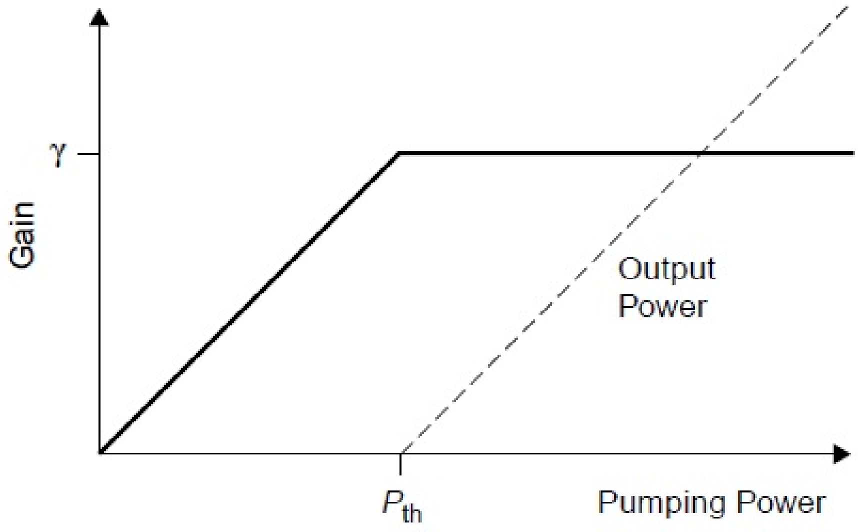

Figure 4.9: Gain versus pumping power.

4.8.3 Threshold Gain

When the laser is operating at steady-state conditions, the net gain in the laser must be 1. If the net gain were greater than 1, the output power would increase. Net gains below 1 cause the output power to drop until the laser ceases to operate.

TO simplify, let us include all the losses except those due to transmission at the mirrors in a effective loss coefficient γ which reduces the effective gain coefficient to $g - \gamma$. We can determine the threshold gain by considering the change in irradiance of a beam of light undergoing a round trip within the laser cavity. We assume that, the laser medium fills the space between the mirrors M_1 and M_2 which have reflectances R_1 and $R_@$ and a separation of L. Then, in travelling from M_1 to M_2, the beam power increases from P_0 to P where,

$$P = P_0 exp(g - \gamma)L. \qquad (4.20)$$

After reflection at M_2, the ebam power will be $R_2 I_0 exp(g - \gamma)L$ and after a complete round trip the final power will be such that the round trip gain G is

$$G = \frac{\text{Final Power}}{\text{Incident Power}} = R_1 R_2\, exp(2(g - \gamma)L). \tag{4.21}$$

As discussed earlier, for lasing condition, $G = 1$ and thus from Equation (4.21), we can write threshold gain g_{th} as

$$g_{\text{th}} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right), \tag{4.22}$$

where the first term represents the volume losses and the second term represents the loss in the form of useful output.

Experimental setup to measure threshold gain: Threshold gain has many practical applications such as determining the minimum required reflectivities of cavity optics as well as allowed insertion losses in the cavity. One can experimentally measure the gain and loss of a lasing medium directly. One simple method (for a heliumneon laser) is to pass the light from an operating heliumneon laser directly through the bore of a bare plasma tube. As the beam from the first laser passes through the plasma tube, it is amplified. By measuring the power of the exiting beam with the bare tube both energized and not energized, the gain of the medium can be measured directly, giving a useful value for g. This is called a MOPA (master oscillator, power amplifier) configuration, the first laser being the oscillator and the second, the amplifier. The experimental setup is shown in Figure 4.10.

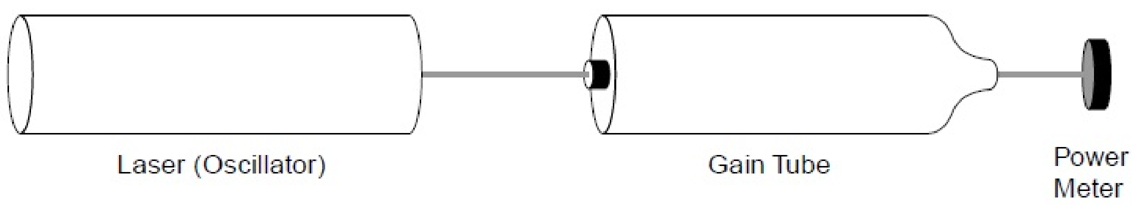

Figure 4.10: Experimental setup for measuring the gain of a laser directly.

A second method for measuring gain is to insert a variable loss into the cavity of a working laser. In its simplest form, this loss may be a slide of glass at a certain angle. At Brewsters angle, for example, there will be zero loss, as least in one polarization. By varying the angle to the point where lasing is extinguished, one may determine the value of the inserted loss. Summing other losses in the laser, such as that of the output coupler (OC), loss at tube windows, and attenuation in the tube itself, one may then calculate the gain of the lasing medium.

Chapter 5

Lasing Transitions and Gain

As discussed in the previous chapter, population inversion is a necessary condition for lasing action. In this chapter, we look closely to the energy states associated with the pumping process for different practical lasers. We will also gain a conception of continuous and pulse mode operation of lasers.

5.1 Selective Pumping

It is evident that the pumping method must ensure that the upper lasing level is populated while the lower level is relatively empty. In the previous chapter, this selectivity of pumping process is described for the He-Ne laser where energy is given to the neon atom through collision with a excited helium atom.

In a solid-state laser such as the Nd:YAG laser, the absorption spectrum in Figure 5.1 shows a large absorption band in the red and near-infrared regions of the spectrum around 750 and 800 nm. High-powered lamps using inert gases (e.g., xenon, krypton) are employed to pump this laser rod optically. Krypton lamps have a rich output in the region where Nd:YAG absorbs, so they are more efficient.

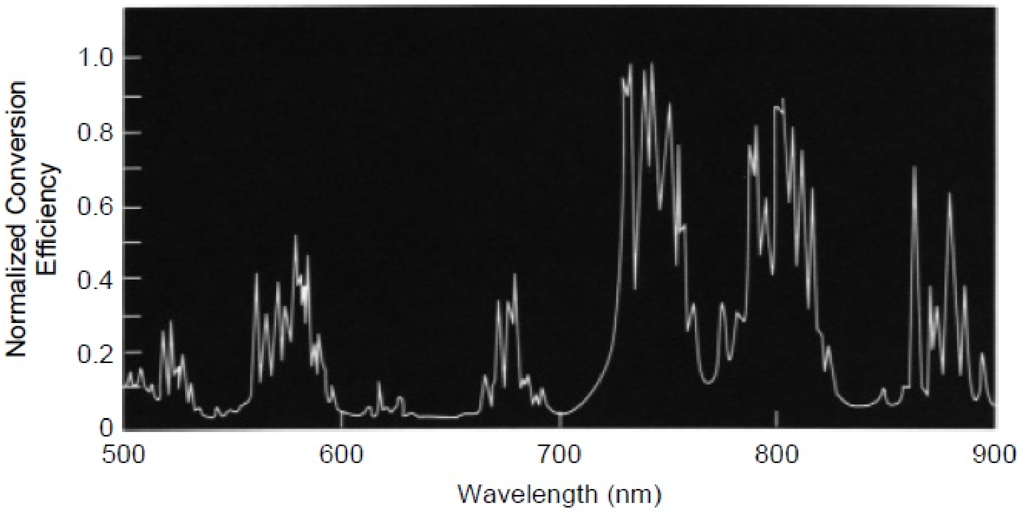

Figure 5.1: Absorption spectrum of Nd:YAG laser.

5.2 Three and Four Level lasers

Lasers are classified by the number of energy levels involved in the actual lasing process as three- or four-level lasers. In a three-level system, energy injected into the gain medium, excites atoms to a pump level above the upper lasing level. From there, atoms decay to the upper lasing level. This decay to the upper lasing level usually occurs by emitting heat, not photons. It is rapid and quickly populates the upper energy level. This upper level often has a long lifetime, so a healthy population of atoms builds in that level. Lasing transitions now occur between the upper level and the ground state, emitting laser light in the process. This system is characterized by the lack of a discrete lower lasing level; the ground state serves that purpose. Figure 5.2(a) shows the energies involved in a three-level laser, including the pump, upper lasing level, and lower-lasing levels.

Three level lasers require very high pump powers because the terminal level of the laser transition is ground state. Thus, more than half of the ground state atoms have to be pumped to the upper state to achieve population inversion. There is also a time delay between the onset of pumping power and lasing output. As pump energy is injected into the three level laser system, a considerable time delay is needed for the buildup of the population in the upper laser level.

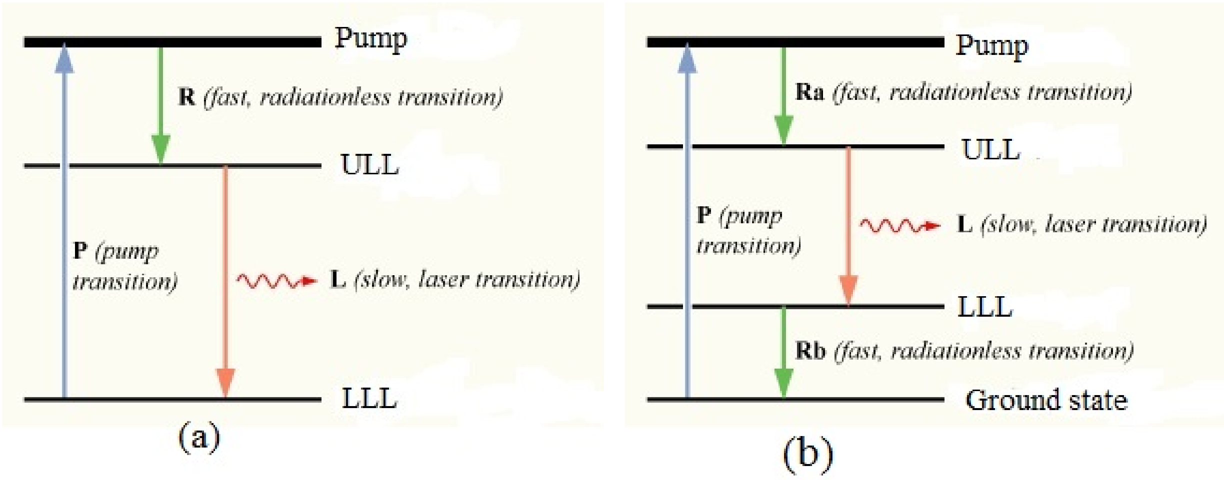

Figure 5.2: Energy states associated with a ideal (a) three level (b) four level laser.

Four-level systems feature a discrete lower lasing level between the upper and ground states. Atoms making a laser transition to the lower state decay further to the ground state, in some cases by emitting a photon. Figure 5.2(b) shows the energy levels involved in an ideal four-level laser.

In a four-level laser, laser gain is realized as soon as pump energy is applied to the system. Pump energy is injected into the pump level, where it decays, in most cases almost instantaneously, to the upper lasing level. Assuming that the upper level has a longer lifetime than the lower level (which is very common in four-level lasers), a population inversion occurs almost immediately after pump energy is injected, although there may not be an output beam until the pump energy exceeds the threshold value. Thus, it is very easy to achieve population inversion is a four-level laser.

5.3 Dynamics of Common Lasers

5.3.1 Ruby Laser

The energy level diagram (Figure 5.3) shows that ruby is basically a three level laser. Pumping is achieved through the absorption of the green and blue spectral regions of a white light discharge. As pump energy is injected into the ruby, one must wait until a significant population of atoms reaches the upper lasing level before lasing can begin.

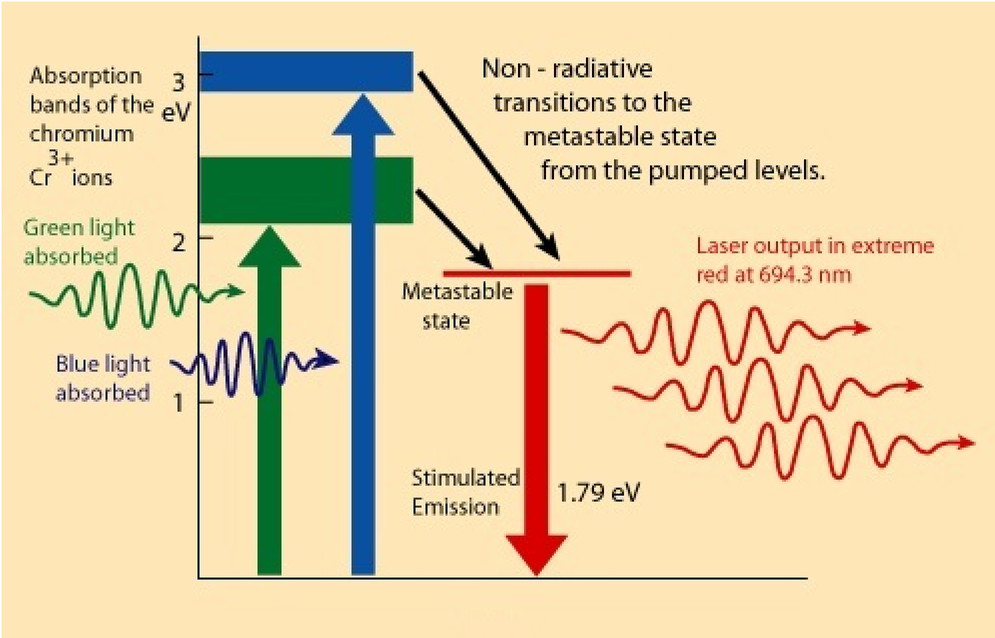

Figure 5.3: Three level ruby laser system.

As a three level laser, ruby is quite efficient. The reasons behind that are the two broad pump bands which readily absorb energy from a flashlamp and the quite long upper laser level lifetime (3 ms) which allows the excited ions to remain there long enough to have a good probability of emitting light by stimulated emission.

5.3.2 Nd:YAG Laser

Nd:YAG laser is a semiconductor laser and also essentially a four level laser as depicted in Figure 5.4. The Nd:YAG system is characterized by a cluster of pump levels from which excited atoms decay rapidly to the upper laser level. There are multiple pump levels, allowing the system to absorb energy at a variety of wavelengths, including the important band at 790 to 810 nm, useful for pumping via a semiconductor laser. All pump levels have short lifetimes, around 100 ns, and decay rapidly to the upper lasing level. The upper level has a very long lifetime of 1.2 ms compared to the lower level, which decays to ground in 30 ns. Many lasing transitions are possible in this system, among which the most powerful one is at 1064 nm.

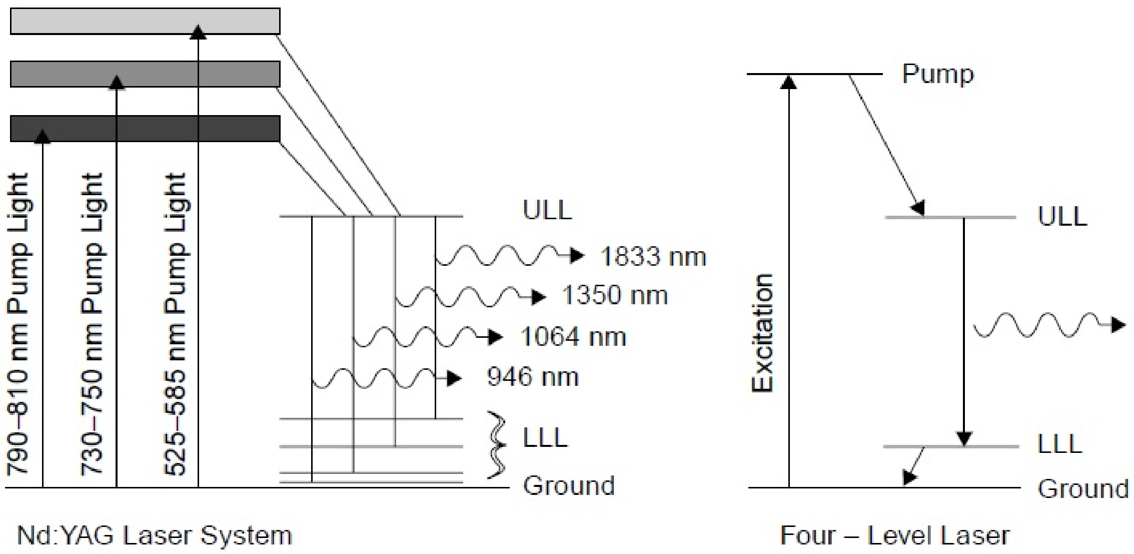

Figure 5.4: Simplified energy diagram for the Nd ion in YAG showing the principle laser transitions.

5.3.3 He-Ne Laser

The He-Ne laser is essentially a four level system as shown in Figure 5.5. The pumping mechanism, involving resonant energy levels between helium and neon, has been described previously, but an examination of the lower levels (of which there are many) reveals an intervening metastable state, to which atoms in one of the lower lasing levels can rapidly decay. This pathway allows depopulation of the lower lasing levels, making population inversion easy to achieve. This rapid decay is radiative; it is accompanied by spontaneous emission of light at about 600 nm. This light does not contribute to laser action but is seen as an orange line in the spectrum of the tube emission as viewed through a spectroscope. Atoms at the lower level must emit the energy somewhere when making the transition to a lower energy level, and the 600-nm spontaneous emission provides that outlet. From that metastable state they must collide with the walls of the tube to return to ground state.

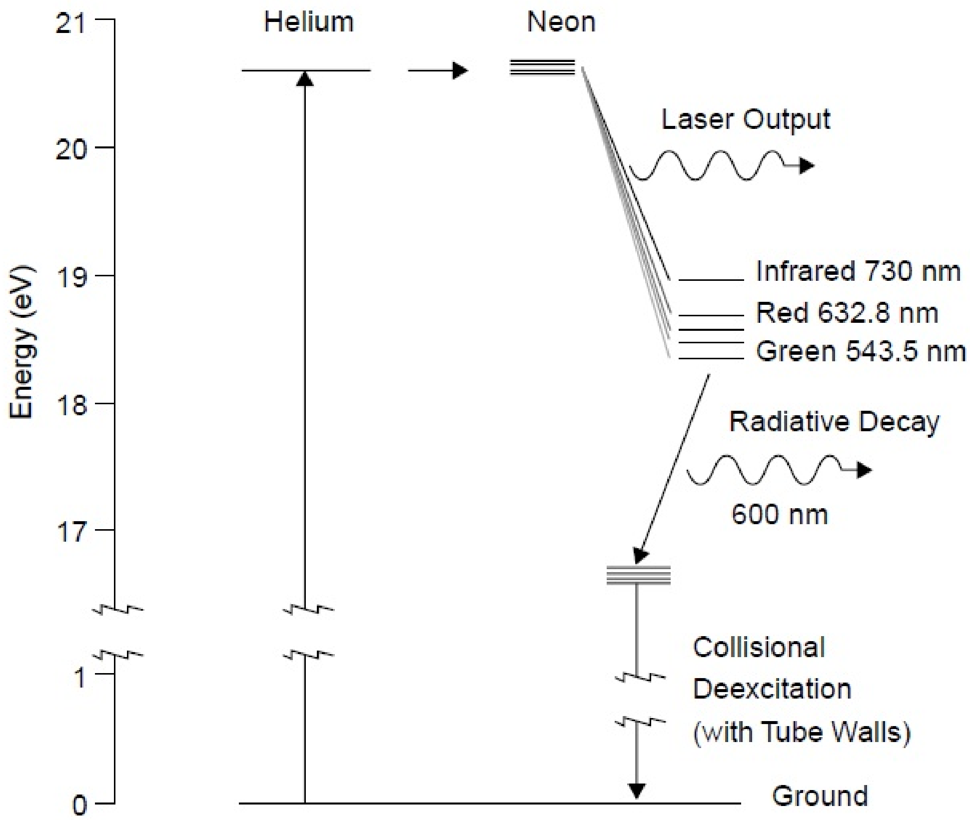

Figure 5.5: Energy levels relevant to the operation of the He-Ne laser.

5.4 CW and Pulsed Lasing Action

In any laser, the probability that a particular system will produce continuous laser output is determined in mainly by the relative lifetime of the levels involved in the laser transition. If the lower level has a relatively long lifetime, atoms in that lower energy state stay there longer, giving them a good chance of absorbing photons as well as of violating the population inversion criteria. In this situation a pulsed laser may still be possible where the upper level is filled quickly and preferentially over the lower level, but eventually the population of the lower level will exceed that of the upper level and lasing will cease. On the other hand, CW operation is possible if the lower level has a short lifetime,and hence the atoms in that state decay quickly to another state, where they will not be available to absorb the newly emitted photons in the laser.

An atom at an upper lasing level, for example, can lose energy by emitting a photon, either spontaneously or through stimulated emission. From Equations (4.5) and (4.4), we can infer that the probability of spontaneous emission decreases with increase of the upper level lifetime (τ) and the probability of stimulated emission increases.

In a four-level laser the ability to lase in CW mode is not only defined by the lifetime of the upper lasing level but also by the lifetime of the lower lasing level. If the lower level has a relatively long lifetime, atoms in that energy state stay there longer, giving them a good chance of absorbing photons. This, of course, would serve to hinder laser action since (1) the laser medium will strongly absorb photons produced by the laser, and (2) this allows the atomic population of the lower level to build violating the required inversion criteria. On the other hand, if the lower level has a short lifetime, atoms in that state decay quickly to another state, where they will not absorb the newly emitted photons in the laser. The latter condition favors CW operation.

N_2 Laser: An example of pulsed laser system

In the nitrogen laser the upper lasing level lifetime (10 ns) is very short compared to the lower

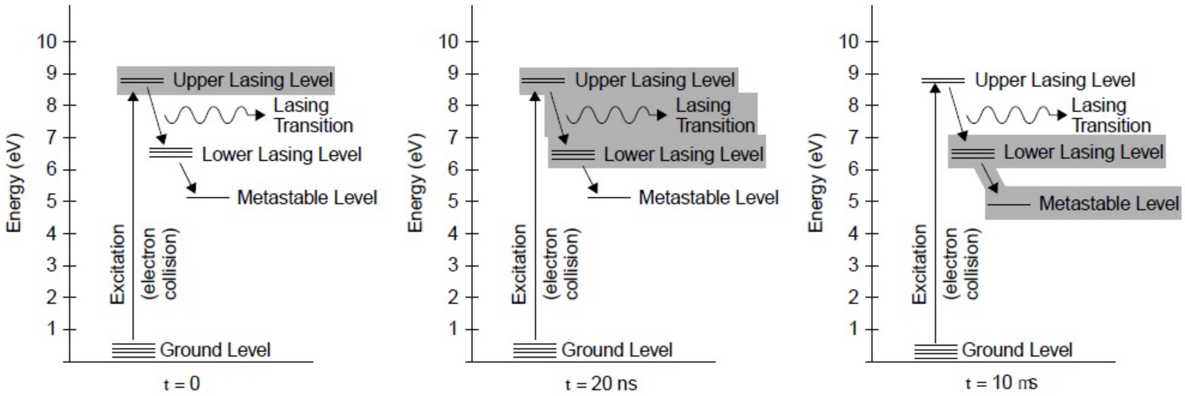

Figure 5.6: Nitrogen laser dynamics.

level (10 ms). If there exists a mechanism to pump energy quickly into the upper level, the laser can operate for a maximum of 10 ns before the lower level is populated, inversion is violated, and lasing ceases. The dynamics of the nitrogen laser are depicted in Figure 5.6, with important energy levels at various times in the lasing process highlighted. At t = 0 s, pumping begins and the upper lasing level fills. Laser action ensues until at about 10 ns, the population of the lower lasing level exceeds that of the upper level and lasing action ceases since population inversion is no longer maintained. About 10 ms later the lower level depopulates via a transition to a metastable energy state that has a longer lifetime yet. The situation also illustrates that it is not possible to operate such a laser in CW mode, since once the lower level has filled, these molecules of nitrogen are no longer available to be pumped to the upper lasing level. The only reason the nitrogen laser operates at all is that a fast pump mechanism exists in which a current of thousands of amperes is made to pass through a small volume of gas. With fast rise times on the current, inversion can be achieved and lasing ensues.

5.5 Thermal Population Effects

The effect of thermal energy on the population of energy levels is negligible in most cases. However, in some laser systems where the lower lasing level is very close to the ground level, thermal effects cannot be ignored. Where two or more transitions are possible, with one having a lower level close to ground state, a transition with a higher lower level may be favored.

For example, in Nd:YAG laser, from Figure 5.4, we see that several lasing transitions are possible, yet 1064 nm is the most powerful and the others rarely appear. As expected, each lower level is actually a cluster of levels tightly clumped together. The lower level for the 946 nm transition is referred to as ground state. Unlike the 1064 nm transition, where the lower level is 1.2 eV above ground state and where the population of this level from temperature alone is negligible, the population of the lower level for the 946-nm transition is very much affected by thermal population effects.

5.6 Depopulation of Lower Energy Levels in Four-Level Lasers

In any four-level laser, some method of depopulation of the lower lasing level exists. These methods may result in the production of a photon of light which do not contribute to lasing action or as the result of a nonradiative processes.

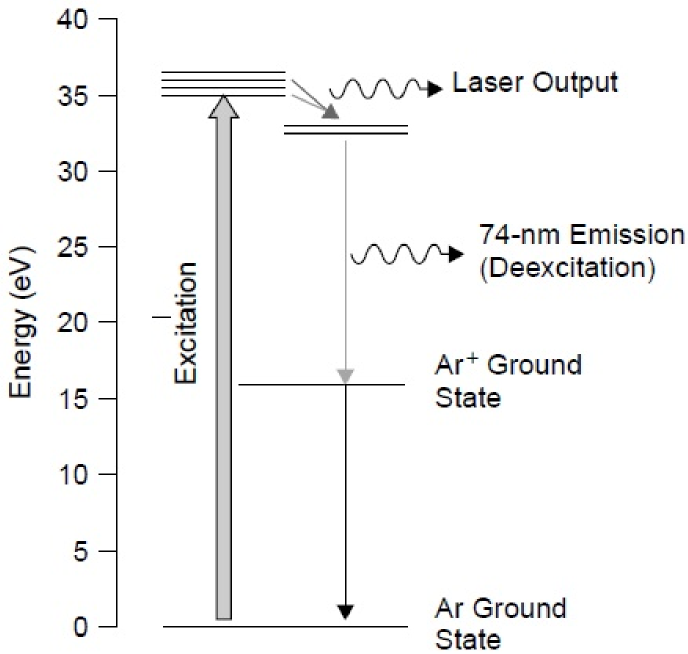

Figure 5.7: Depopulation of the argon-ion lower lasing level.

In the argon laser depopulation occurs via radiative energy loss. Here, argon ions at the lower lasing levels drop to a lower level by emission of a extreme- UV photon as shown in Figure 5.7. This photon does not contribute to laser action, nor is it normally observed as a spontaneous emission from the tube since few materials (including quartz windows) are transparent to this extremely short wavelength.

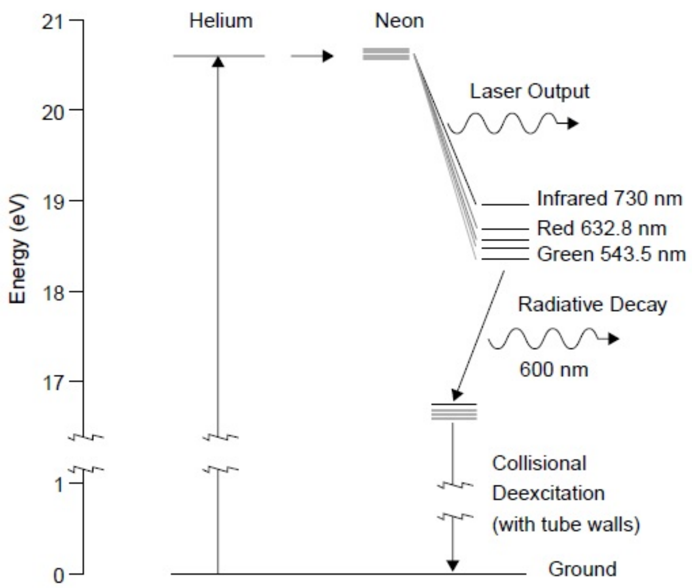

Figure 5.8: Depopulation of neon atoms in the He-Ne laser.

Atoms of neon in a He-Ne laser undergo a two-stage process, outlined in Figure 5.8, to relax to the ground state, from where they can again be pumped to the upper lasing level. From the lower lasing level neon atoms first undergo a radiative transition to an intermediate level by emission of a photon around 600 nm. From that intermediate level, radiative emission is not possible, due to quantum-mechanical selection rules for allowed transitions, so the only avenue available is collisional transfer of energy between the excited neon atoms and the walls of the laser tube.

Depopulation occurs in semiconductor lasers through non radiative transitions like emission of a phonon.

5.7 Rate Equation Analysis for Atomic Transitions

Let us define the cross section of an atom, which specifies the effective size of an atom. Cross section is important because it gives rise to *transitional probability* which is denoted by W. For a given transition between two energy levels, W represents the odds that a particular atom will make a particular transition.

The rate of energy flow into or out of a particular level is the change in population of the level (ΔN) over a given time period (Δt). It can be expressed as

$$r = \frac{dN}{dt} = WN, \tag{5.1}$$

where W is the probability of a transition and N is the population of the level.

It must be noted that W is a function of the intensity of incident light. For an absorption process it is obvious that more light intensity results in a higher probability of absorption of a photon. The same holds true for stimulated emissions, where a higher intensity of light results in a higher probability of a stimulated emission occurring. The probability of emission or absorption is related to the cross section by

$$W = \frac{\sigma I}{h\nu}, \tag{5.2}$$

where σ is the cross section of the transition, and I the intensity of the incident photon stream.

We can also relate the probability of transition to the *Einstein's coefficient* B_{12} as

$$W = B_{12}\rho, \tag{5.3}$$

where ρ is the energy density.

Let us consider a two level atomic system as depicted in Figure 5.9. For the upper level:

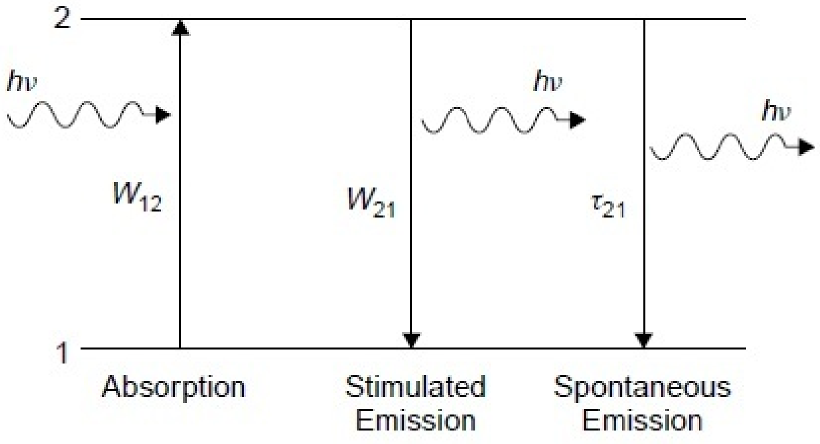

Figure 5.9: Transitions in a two-level atomic system.

$$\frac{dN_2}{dt} = (\text{absorptionfrom}1 \rightarrow 2) - (stimulatedemissionfrom2 \rightarrow 1) - (spontaneousdecay2 \rightarrow 1). \tag{5.4}$$

Or,

$$\frac{dN_2}{dt} = W_{12}N_1(t) - W_{21}N_2(t) - \frac{N_2(t)}{\tau_{21}} \tag{5.5}$$

Similarly, for the lower level:

$$\frac{dN_1}{dt} = W_{21}N_2(t) - W_{12}N_1(t) + \frac{N_2(t)}{\tau_{21}} \tag{5.6}$$

We can now solve for the rate of change of the population difference as

$$\frac{d\Delta N(t)}{dt} = \frac{dN_1(t)}{dt} - \frac{dN_2(t)}{dt}; = -2W_{12}N_1(t) + 22W_{21}N_2(t) + \frac{2N_2(t)}{\tau_{21}} \tag{5.7}$$

But the probability of an absorption is the same as the probability of a stimulated emission (i.e.,$W_{12} = W_{21}$), so that we may now express the equation as

$$\frac{d\Delta N(t)}{dt} = -2W_{21}\Delta N(t) + \frac{2N_2(t)}{\tau_{21}} \tag{5.8}$$

Furthermore, the total population of the system (N_0) remains constant with $N_0 = N_1(t) + N_2(t)$, so we may substitute $\Delta N(t) = N_0; - 2N_2(t)$. Here, a decrease of one atom from N_2 results in an increase of one atom in level N_1, so that the total change in the difference ΔN is two atoms. At steady state, the rate of change of population difference is zero, so equation (5.8) may be solved as

$$\Delta N(t) = \frac{N_0}{1 + 2W_{21}\tau_{21}} \tag{5.9}$$

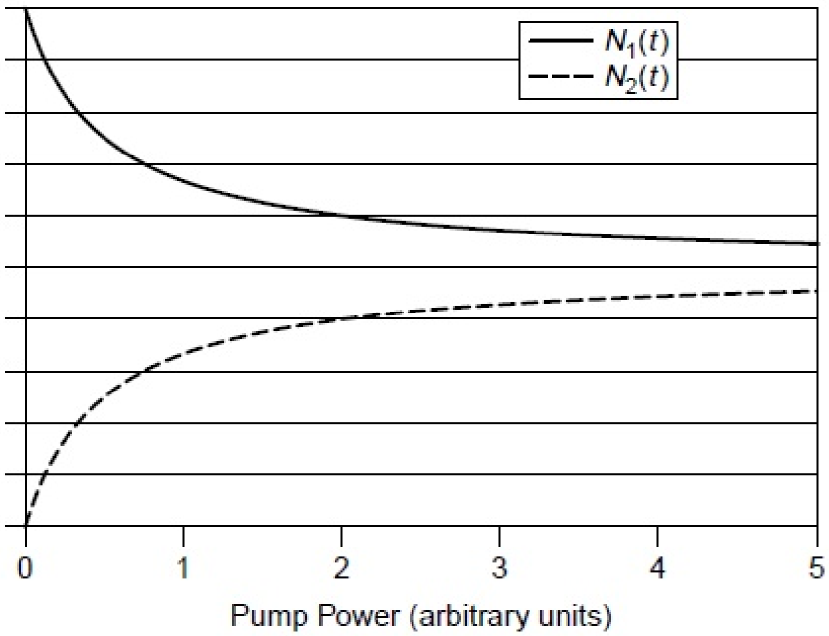

Figure 5.10: Energy-level populations for a two-level system.

The atomic population of each level at various pump power levels is plotted in Figure 5.10, with N_1 starting at N_0 at zero pump power. In other words, there is no population at level N_2; in reality, N2 would be thermally populated according to Boltzmann statistics, but for most systems this population would be negligible. Under no conditions can an inversion be achieved in this two-level system, so laser action is not possible with a two-level atomic system.

5.8 Rate Equation Analysis for Three and Four Level System

Most practical lasers feature three- and four-level atomic systems, so we shall apply the same technique used in the analysis of the two-level system to these systems.

5.8.1 Three Level Laser

The pump level is populated via upward transitions from the ground state and loses population via downward decay to the ULL. The rate equation for the pump level can be expressed as the change of population of atoms in the pump level as follows:

$$\frac{dN_3}{dt} = W_{13}(N_1 - N_3) - \frac{N_3}{\tau_3} \tag{5.10}$$

where $W_1 3$ is the probability of an atom making the transition from level 1 to level 3 and τ_3 is the decay lifetime of the pump level. The decay lifetime τ_3 is a total lifetime representing both the decay from level 3 to level 2 and the decay from level 3 to level 1. If each decay path $3 \rightarrow 2$ and $3 \rightarrow 1$ has time constants τ_{32} and τ_{31}, respectively, then, $1/\tau_3 = 1/\tau_{32} + 1/\tau_{31}$.

Following a similar line of reasoning, the rate equation for the upper lasing level can be expressed as

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}. \tag{5.11}$$

Under steady state condition, the rates become zero. Thus equating equation (5.10) to zero, we get,

$$W_{13}N_1 = N_3\left(\frac{1}{\tau_3} + W_{13}\right). \tag{5.12}$$

This may be simplified mathematically, since τ_3 is a small quantity and hence, numerically speaking, $1/\tau_3$ is much greater that W_{13} so that the latter term can essentially be ignored, giving us an expression for N_3:

$$W_{13}\tau_3 N_1 = N_3. \tag{5.13}$$

Thus, the population of the pump level depends on the rate of pumping itself and on the total lifetime of that level.

We now equate equation (**??**) to zero to get

$$N_2 = N_3 \frac{\tau_{21}}{\tau_{32}} \tag{5.14}$$

Finally, combining equations (5.13) and (5.14) and simplifying equation (5.14) by assuming that $\tau_{32} = \tau_3$ (i.e., assuming that there is no leakage from the pump level to ground and that all atoms in the pump level decay solely to the ULL), we yield an expression for inversion now defined as $\Delta N = N_2(t) - N_1(t)$ as follows:

$$\Delta N = N_1(W_{13}\tau_{21} - 1) \tag{5.15}$$

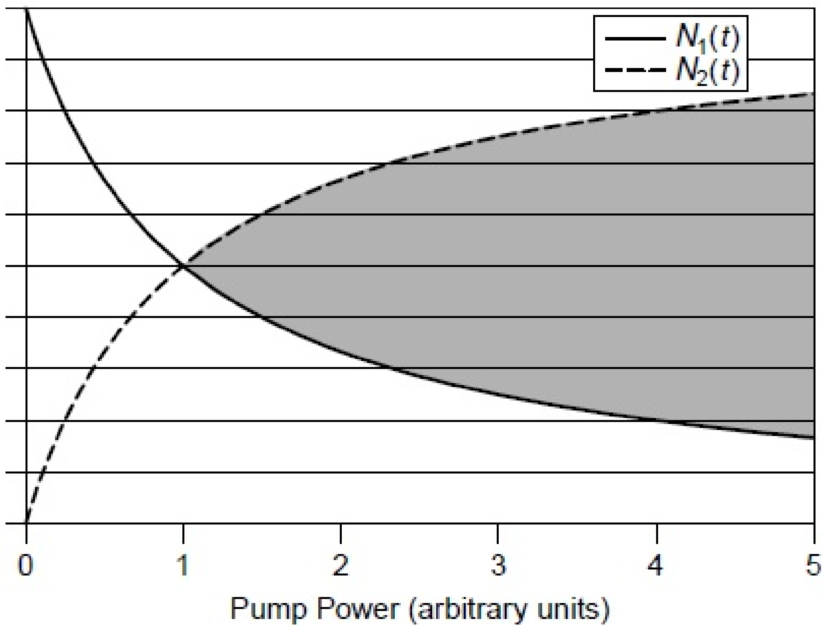

Figure 5.11: Energy-level populations for a three-level system.

In a three level laser, inversion does not occur with the onset of pumping. A minimum pumping rate equal to $W_{13}\tau_{21}$ (where W_{13} is proportional to pumping rate) is needed just to get half of the population at ground state to the upper lasing level. The population of the upper and lower lasing levels is plotted in Figure , with inversion indicated in the shaded area.

5.8.2 Four Level Laser

The rate equation for the pump level can be expressed as the change of population of atoms in the pump level as follows:

$$\frac{dN_4}{dt} = W_{14}(N_1 - N_4) \; - \; \frac{N_4}{\tau_4}, \tag{5.16}$$

where W_14 is the probability of an atom making the transition from level 1 to level 4 and τ_4 is the total lifetime of the pump level.

The equation for the upper lasing level can be expressed as

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} \; - \; \frac{N_3}{\tau_3}. \tag{5.17}$$

The rate equation for the lower lasing level is

$$\frac{dN_2}{dt} = \frac{N_4}{\tau_{42}} \; + \; \frac{N_3}{\tau_{32}} \; - \; \frac{N_2}{\tau_{21}}. \tag{5.18}$$

For steady state condition, we equate the rate for pump level to zero. Thus, from equation (5.16), we get,

$$W_{14}(N_1 - N_4) - \frac{N_4}{\tau_4} = 0. \tag{5.19}$$

But the population of N_4 in a real laser will be much smaller than that of N_1, so we can simply alter equation (5.19) by eliminating one term from the left side of the equation to get:

$$W_{14}\tau_4 N_1 = N_4. \tag{5.20}$$

Equation (5.20) states that as long as the excitation is not large enough to completely drive a large proportion of ground-state atoms to higher-energy states, the population of the pump level is proportional to the pump rate.

Similarly, for the ULL, we get,

$$N_3 = N_4 \frac{\tau_3}{\tau_{43}} \tag{5.21}$$

Now, we equate the rate of the LLL to zero to get

$$N_2 = N_3 \frac{\tau_{21}}{\tau_{32}}. \tag{5.22}$$

Finally we combine the equations (5.19), (5.21) and (5.22), to derive the expression for population inversion as follows:

$$\Delta N(t) = N_3(t) - N_2(t) = W_{14} N_1 \tau_3 - W_{14} N_1 \tau_{21} \tag{5.23}$$

But in a practical laser the decay time from the lower lasing level ($2 \rightarrow 1$) is very fast, so this term is insignificant against the first term, yielding the result

$$\Delta N(t) = W_{14} N_1 \tau_3. \tag{5.24}$$

In other words, a population inversion results when any pump energy is supplied, as plotted in Figure 5.12, where the inversion is indicated as the shaded area.

5.9 Gain Revisited

We can express the gain coefficient of the system as proportional to the population inversion according to

$$g = (N_2 - N_1)\sigma, \tag{5.25}$$

where $(N_2 - N_1))$ is the population inversion and σ is the cross section of stimulated emission process. Cross section can be expressed mathematically as the product of the transition strength (or oscillator strength), S, and the lineshape function of gain, g(ν), which gives the normalized gain of the stimulated emission process at any given frequency. Transition strength is related to the spontaneous lifetime of the atomic species, so we may express cross section mathematically

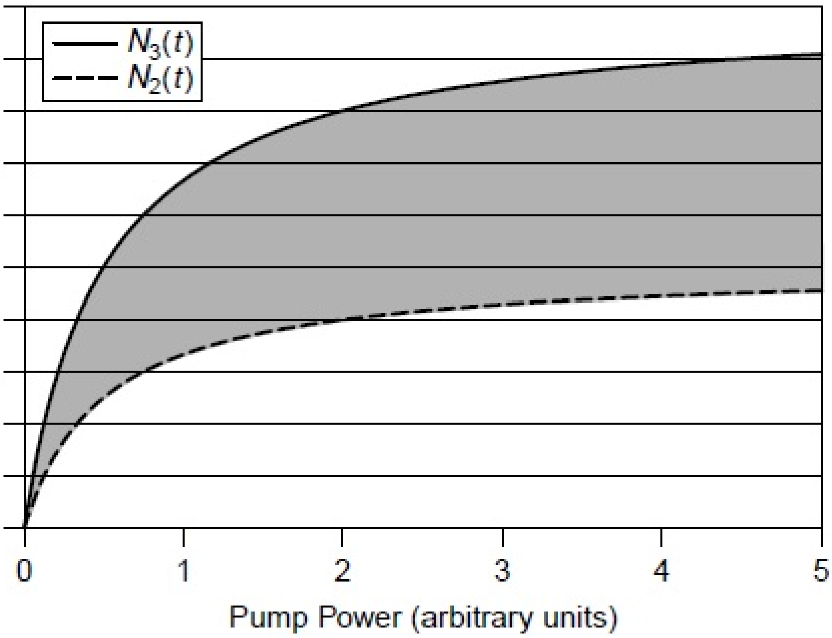

Figure 5.12: Energy-level populations for a four-level system.

as

$$\sigma(\nu) = Sg(\nu) = \frac{\lambda^2}{8\pi t_{\mathrm{sp}}} g(\nu) \tag{5.26}$$

where $g(\nu)$ is the lineshape of the gain function and t_{sp} is the spontaneous lifetime of the species.

The laser emission is not infinitely sharp, it has finite linewidth; and thus, the gain spreads out over a range of frequencies. The lineshape of the gain function, for many lasers, is approximately Gaussian in shape which can be expressed mathematically as

$$g(\lambda) = g(\lambda_0)\; exp\left[-\frac{(\lambda - \lambda_0)^2}{2\delta^2}\right] \tag{5.27}$$

where δ is the FWHM width of the gain spectrum, which models, mathematically, the Gaussian shape of a typical gain curve. For many purposes it is sufficient to use a gain value at the maxima on the curve, at the center wavelength or frequency (ν_0). A reasonable approximation of the gain curve for homogeneous broadening mechanisms is:

$$g(\nu_0) = \frac{2}{\pi \Delta \nu}. \tag{5.28}$$

For gas lasers, broadening mechanisms like Doppler broadening is inhomogeneous and the approximation of the gain lineshape is

$$g(\nu_0) = \frac{1}{\Delta\nu}, \tag{5.29}$$

where $\Delta\nu$ is the linewidth (FWHM) of the transition.

By substituting these approximations for the gain lineshape into equation (5.26), we can calculate the maximum value of the cross section to yield σ_0. Finally, using σ_0 in equation (5.25), we may calculate the gain of the laser for a given inversion.

We derived the equation of threshold gain in Chapter. 4 (equation (4.16)). By utilizing this with equation (5.25), we get an answer for threshold in terms of the population inversion required to allow a laser to oscillate as follows:

$$\Delta N_{th} = \frac{2\gamma l + \ln(1/R_1 R_2)}{\sigma_0 l}. \tag{5.30}$$

The numerator term in the equation represents round-trip losses, including absorption by the laser medium itself ($2\gamma l$) as well as losses at both cavity mirrors [$\ln(1/R_1 R_2)$].

5.10 Saturation

The rate equation analysis given in Section 5.7 is a steady-state analysis assuming no radiation in the cavity. If radiation is present in the cavity, stimulated emission will occur at the rate of $N_2 W_{21}$, and this will represent a loss. There will also be a process of absorption at a rate of $N_1 W_{12}$ in the opposite direction as photons are absorbed and the energy is used to pump atoms in the LLL back to the ULL. Rate equations must now be rewritten to include these new terms, and it will be noted that as the photon flux increases, the amount of inversion as well as gain will decrease.

In a laser, gain is not a constant value but rather varies with incident power. Imagining the laser gain medium as an amplifier, the gain is quite large. This is termed the small-signal value, in which the ULL is well populated and is replenished continually by pumping, so that the population does not change appreciably. As the input signal to the amplifier reaches a large value, the photon flux is large enough to depopulate the ULL, which then lowers the overall gain in the device since fewer excited atoms will be available at that level to contribute to the stimulated emission process. This results in a saturated gain figure in which the gain is reduced by large photon fluxes in the cavity.

We recall from Section 4.6 that gain is an increase in power for a given distance traveled through the laser amplifier medium. In a normal laser amplifier we would expect power to increase exponentially with length according to

$$P_{\text{output}} = P_{\text{input}} \exp(g_0 l), \tag{5.31}$$

where g_0 is the small-signal gain of the laser amplifier and l is the length of the amplifier. This represents the maximum gain that a laser amplifier can deliver, but as the amplifier saturates, eventually the power increase is a linear function of power according to

$$P_{\text{output}} = P_{\text{input}} + g_0 l. \tag{5.32}$$

The gain of a saturated system is obviously dependent on the photon flux inside the cavity since it is these photons that generate the stimulated emissions which serve to deplete the ULL and hence decrease gain.The saturated gain is then

$$g_{sat} = \frac{g_0}{1 + \frac{\rho}{\rho_{sat}}} \tag{5.33}$$

where g_0 is the unsaturated gain of the amplifier, ρ the photon flux in the system, and ρ_{sat} is called the saturation flux.

5.11 Required Pump Power and Efficiency

5.11.1 Pump Power

The minimum pump energy that must be delivered to the system:

$$P_{\text{minimum}} = \frac{dN_{ULL}}{dt} Vh\nu_{mp} \tag{5.34}$$

where V is the active volume of the amplifier and $h\nu_{mp}$ is the photon energy required at the ULL. The rate dN_{ULL}/dt in the equation is the rate at which energy decays from the pump level to the upper lasing level and so represents the energy flowing into the ULL from the pump level above. A substitution for this rate may be determined by solving the equation for decay of the level, which expresses the population of the level at any time t as

$$N_{ULL}(t) = N_{ULL}(0)exp(-\frac{t}{\tau}) \tag{5.35}$$

where $N_{ULL}(0)$ is the population of the ULL at time $t = 0$ and τ is the spontaneous lifetime of the level. Taking the differential of the equation with respect to time yields an answer of

$$\frac{dN_{ULL}}{dt} = -\frac{1}{\tau}N_{ULL} \tag{5.36}$$

Let us consider the threshold population inversion as stated in equation (5.30). Knowing the reflectivities of both cavity mirrors as well as loss in the lasing medium, we may calculate the threshold population inversion and further equate this to N_2 using the approximation stated. Furthermore, by determining the spontaneous lifetime of the lasing species, we may solve the required rate of transitions from the ULL to the LLL (i.e., dN_{ULL}/dt). Finally, computation of the volume and photon energy of the system allows solution of the minimum required pump power.

5.11.2 Efficiency

In a real laser, 100% of the pump light does not translate into pump energy for the laser. Most gas lasers have overall efficiencies of well under 1%. Reasons for low efficiency include the efficiency of the pump source itself at converting electrical energy into optical output, the efficiency with which pump light is coupled into the lasing medium, the absorption efficiency of the lasing material, and the quantum efficiency of the atomic system.

- The efficiency of conversion of electrical energy into optical output (denoted η_{optical}) depends on the lamp technology employed. Use of a laser diode as an optical pump can lead to conversion efficiencies of up to 50%, while the use of halogen lamps results in poor efficiency since the lamp itself converts at most 10% of the electrical input power to visible light.

- Coupling of the pump light to the laser medium (denoted η_{coupling}) is highly dependent on the geometry of the laser medium.

- Absorption efficiency (denoted $\eta_{\text{absorption}}$) of the medium varies with the wavelength of the pump source, as have seen in Section 5.1. A laser medium with broad absorption bands is desirable since it can absorb more pump energy from broadband sources such as lamps.

- Finally, the intrinsic quantum efficiency (denoted η_{quantum}) of the atomic system affects the overall efficiency of the laser. It can be given as

  $$\eta_{\text{quantum}} = \frac{E_{ULL} - E_{Ground}}{E_{Pump} - E_{Ground}}. \tag{5.37}$$

  It represents the energy difference between the pump and ULL and is completely dependent on the particular atomic system.

The overall pump efficiency of the laser is a product of all four efficiencies:

$$\eta_{\text{pump}} = \eta_{\text{optical}} \times \eta_{\text{coupling}} \times \eta_{\text{absorption}} \times \eta_{\text{quantum}} \tag{5.38}$$

5.12 Output Power

For the threshold condition, gain must be equal to loss. However, if gain exceeds the threshold value, it must saturate down to reach an equilibrium point once again. The intensity of light inside the cavity grows in the process, and a usable output beam appears as a fraction of that intracavity intensity. If we equate threshold gain and the saturated gain, we get the intensity of light inside the laser cavity, I as

$$I = \frac{[2g_0/(2\gamma l + \ln(1/R_1R_2)) - 1]I_{sat}}{2} \tag{5.39}$$

Thus, we can calculate the output power intensity in the laser medium.