coils a_1 and a_2 can be represented by phasor OX in Fig. B.1, and that of coils a_3 and a_4 by the phasor OY. The time-phase displacement between these two voltages is the same as the electrical angle between adjacent slots, so that OX and OY coincide with the centerlines of adjacent slots. The resultant phasor OZ for phase a is obviously smaller than the arithmetic sum of OX and OY.

In addition, the effect of fractional pitch in Fig. B.2 is that a coil links a smaller portion of the total pole flux than if it were a full-pitch coil. The effect can be superimposed on that of distributing the winding by regarding coil sides a_2 and $-a_1$ as an equivalent coil with the phasor voltage OW (Fig. B.2), coil sides a_1, a_4, $-a_2$, and $-a_3$ as two equivalent coils with the phasor voltage OX (twice the length of OW), and coil sides a_3 and $-a_4$ as an equivalent coil with phasor voltage OY. The resultant phasor OZ for phase a is obviously smaller than the arithmetic sum of OW, OX, and OY and is also smaller than OZ in Fig. B.1.

The combination of these two effects can be included in a *winding factor* k_w to be used as a reduction factor in Eq. B.1. Thus, the generated voltage per phase is

$$E = \sqrt{2}\pi k_w f N_{ph} \Phi \tag{B.2}$$

where N_{ph} is the total turns in series per phase and k_w accounts for the departure from the concentrated full-pitch case. For a three-phase machine, Eq. B.2 yields the line-to-line voltage for a Δ-connected winding and the line-to-neutral voltage for a Y-connected winding. As in any balanced Y connection, the line-to-line voltage of the latter winding is $\sqrt{3}$ times the line-to-neutral voltage.

B.1.2 Breadth and Pitch Factors

By separately considering the effects of distributing and of chording the winding, reduction factors can be obtained in generalized form convenient for quantitative analysis. The effect of distributing the winding in n slots per phase belt is to yield n voltage phasors displaced in phase by the electrical angle γ between slots, γ being equal to 180 electrical degrees divided by the number of slots per pole. Such a group of phasors is shown in Fig. B.3a and, in a more convenient form for addition, again in Fig. B.3b. Each phasor AB, BC, and CD is the chord of a circle with center at O and subtends the angle γ at the center. The phasor sum AD subtends the angle $n\gamma$, which, as noted previously, is 60 electrical degrees for the normal, uniformly distributed three-phase machine and 90 electrical degrees for the corresponding two-phase machine. From triangles OAa and OAd, respectively,

$$OA = \frac{Aa}{\sin(\gamma/2)} = \frac{AB}{2\sin(\gamma/2)} \tag{B.3}$$

$$OA = \frac{Ad}{\sin(n\gamma/2)} = \frac{AD}{2\sin(n\gamma/2)} \tag{B.4}$$

Equating these two values of OA yields

$$AD = AB\,\frac{\sin(n\gamma/2)}{\sin(\gamma/2)} \tag{B.5}$$

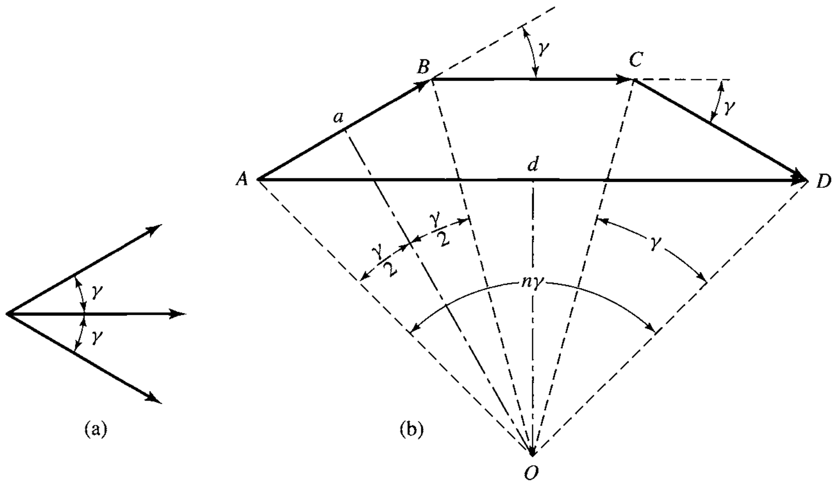

Figure B.3 (a) Coil voltage phasors and (b) phasor sum.

But the arithmetic sum of the phasors is $n(AB)$. Consequently, the reduction factor arising from distributing the winding is

$$k_b = \frac{AD}{nAB} = \frac{\sin(n\gamma/2)}{n\sin(\gamma/2)} \tag{B.6}$$

The factor k_b is called the *breadth factor* of the winding.

The effect of chording on the coil voltage can be obtained by first determining the flux linkages with the fractional-pitch coil. Since there are n coils per phase and N_{ph} total series turns per phase, each coil will have $N = N_{ph}/n$ turns per coil. From Fig. B.4 coil side $-a$ is only ρ electrical degrees from side a instead of the full 180°. The flux linkages with the N-turn coil are

$$\lambda = NB_{peak}lr\left(\frac{2}{\text{poles}}\right)\int_{\rho+\alpha}^{\alpha}\sin\theta\, d\theta \tag{B.7}$$

$$\lambda = NB_{peak}lr\left(\frac{2}{\text{poles}}\right)[\cos(\alpha+\rho) - \cos\alpha] \tag{B.8}$$

where

l = axial length of coil side
r = coil radius
poles = number of poles

With α replaced by ωt to indicate rotation at ω electrical radians per second, Eq. B.8 becomes

$$\lambda = NB_{peak}lr\left(\frac{2}{\text{poles}}\right)[\cos(\omega t+\rho) - \cos\omega t] \tag{B.9}$$